

# KINEMATICS

## 1. Basic quantities

This is the area of physics is concerned with motion.

### FRAME OF REFERENCE

is a set of bodies or points related to which we state if some object is at rest or in motion. In fact nothing is at absolute rest, because motion is one of the basic attributes of matter. To simplify situations and to be able to solve some examples and calculations we usually connect the frame of reference with .....

### MASS POINT

is used to replace the real object in physical examples in order to simplify them. It has the same mass as the object and it is placed in the centre of mass of the object. Because we have neglected the size of the 3D object, we can disregard the friction force and air resistance, for example. Also, forces acting on the mass point cannot cause rotation.

### CENTRE OF MASS

is the point of action of the force of gravity, which attracts the object towards the centre of the Earth.

### TRAJECTORY

is the line which describes the path of the object. It can be a straight line, a curve, a part of a circle,... It relates to the chosen frame of reference. It is not a physical quantity, because we cannot measure it. We just describe it.

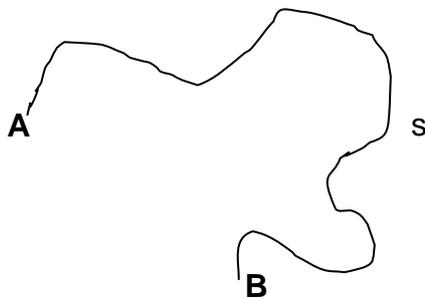
### DISTANCE ( $s, l, a, b, c, x, y, z$ )

is the length of the ..... We measure it in..... It is a scalar/vector.

### DISPLACEMENT ( $\vec{d}, d$ )

is a vector quantity representing the change of position of the object or mass point. Its size is the distance between the first and the last point of the trajectory. The direction is from the first to the last point of the trajectory regardless of changes in direction during the motion. It is considered to be a straight line from A to B.

Ex.:



*A is the first and B is the last point of the trajectory*

Find the displacement and finish the equation/inequality

$$s \quad \left| \vec{d} \right|$$

### SPEED ( $v$ )

its basic unit is ..... . It is a scalar quantity which would tell you how far in metres the object will travel or the mass point will move in one second. It doesn't measure the direction of the motion. Another unit used very often is  $\text{km}\cdot\text{h}^{-1}$ .

$$v = \frac{s}{t}$$

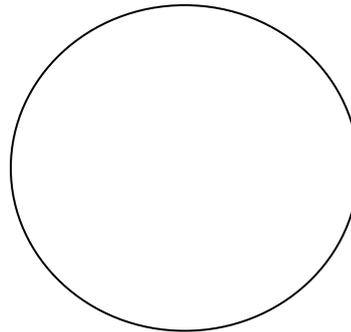
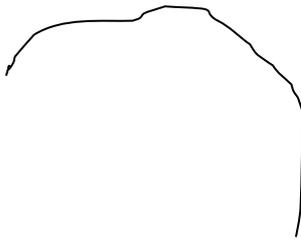
average speed  $\bar{v} = \frac{\text{total distance travelled}}{\text{total time taken}}$

### VELOCITY ( $\vec{v}$ , $v$ )

is also measured in  $\text{m}\cdot\text{s}^{-1}$  or  $\text{km}\cdot\text{h}^{-1}$ . Its direction is always tangential to the trajectory of the object or mass point.

$$\vec{v} = \frac{d\vec{d}}{dt} = \frac{\Delta\vec{d}}{\Delta t} = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}$$

Sketch the velocity vectors from different points on the following trajectories:



VOCABULARY: NEGLECT

ASSUME

NEGLIGIBLE

ASSUMPTION

### TYPES OF MOTION

- according to speed

\* steady, uniform  $v = \text{const.}$

for any time intervals the distance travelled is the same

\* non-uniform  $v \neq \text{const.}$

steadily accelerated m. – special type  $\frac{\Delta v}{\Delta t} = \text{const.}$

- according to trajectory

\* linear, circular

## 2. Steady or uniform linear motion

is the simplest type of motion, where the size and direction of the velocity does not change.

The motion must have initial speed  $v_0 = v = \text{const}$

### Questions:

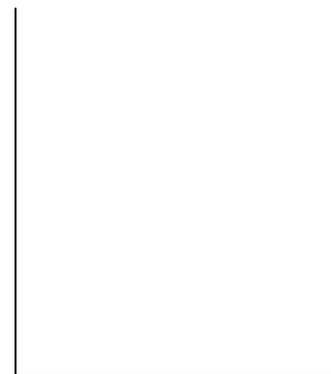
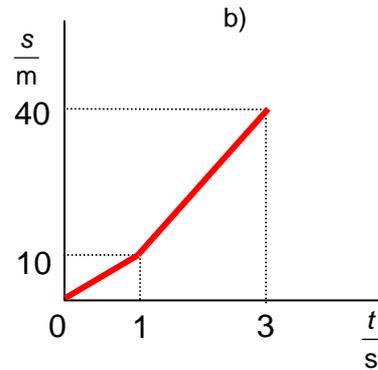
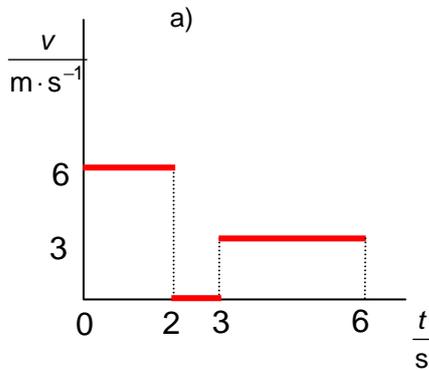
1. A car moves at  $60 \text{ km}\cdot\text{h}^{-1}$ .

- What is the distance travelled during 20 minutes?
- Sketch distance-time ( $s-t$ ), speed-time ( $v-t$ ) and acceleration-time ( $a-t$ ) graphs.
- Discuss what the area under  $v-t$  graph represents.

2. Assume a motorcycle moving steadily in a straight direction at  $72 \text{ km}\cdot\text{h}^{-1}$ .

- Calculate the distance travelled in 10 seconds.
- Draw  $s-t$  and  $v-t$  graphs.

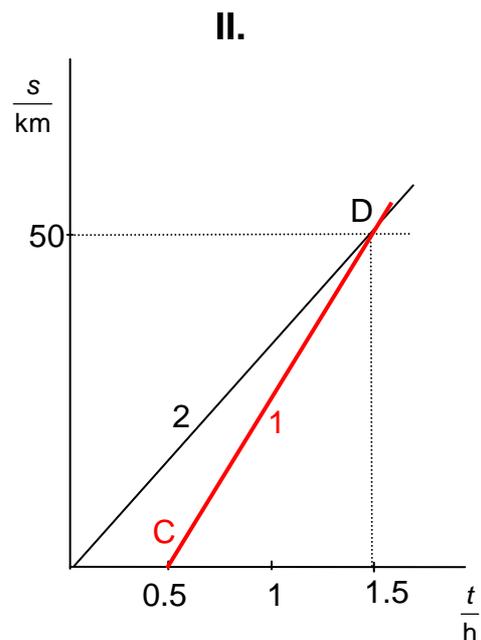
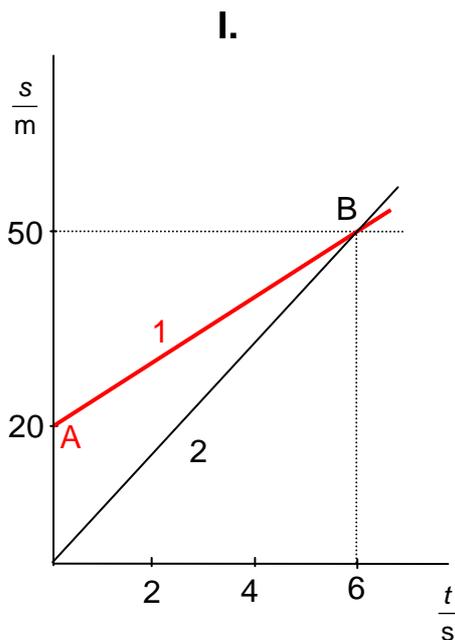
3. Sketch appropriate  $s-t$  ( $v-t$ ) graph for a given  $v-t$  ( $s-t$ ) graph



4. A car moves for 1 hour at  $50 \text{ km}\cdot\text{h}^{-1}$  and 45 minutes at  $20 \text{ m}\cdot\text{s}^{-1}$ . Calculate its average speed.

5. A car moves  $\frac{3}{4}$  of the total time at  $54 \text{ km}\cdot\text{h}^{-1}$  and the rest of the time at  $72 \text{ km}\cdot\text{h}^{-1}$ . Calculate its average speed.

6. A car moves  $\frac{3}{4}$  of the total distance at  $54 \text{ km}\cdot\text{h}^{-1}$  and the rest of the distance at  $72 \text{ km}\cdot\text{h}^{-1}$ . Calculate its average speed.
7. A man cycles to his destination with the wind at  $8 \text{ m}\cdot\text{s}^{-1}$  and then back against the wind at  $4 \text{ m}\cdot\text{s}^{-1}$ . What is his average speed?
8. A body travels from A to B in 10 minutes with a speed of  $50 \text{ km}\cdot\text{h}^{-1}$  and returns with a speed of  $110 \text{ km}\cdot\text{h}^{-1}$ . Find the average speed in  $\text{m}\cdot\text{s}^{-1}$  and in  $\text{km}\cdot\text{h}^{-1}$ . Sketch the v-t and s-t graphs.
9. Peter goes for two hours at  $6 \text{ km}\cdot\text{h}^{-1}$  and then he climbs a steep hill for one hour at  $3 \text{ km}\cdot\text{h}^{-1}$ . What is his average speed? Sketch the v-t and s-t graphs.
10. A car goes  $\frac{1}{4}$  of the whole distance at  $90 \text{ km}\cdot\text{h}^{-1}$  and the rest at  $75 \text{ km}\cdot\text{h}^{-1}$ . Find its average speed in  $\text{m}\cdot\text{s}^{-1}$  and in  $\text{km}\cdot\text{h}^{-1}$ . Sketch the v-t graph.
11. The graphs I. and II. represent two different examples of motion of two cars 1 and 2. For each example calculate
- the speeds of the cars 1 and 2
  - the physical meaning of points A, B, C and D.
  - How far and when will the cars meet?



12. A police car P at a distance of 800 m from a crossroad is approaching the crossroad at the uniform speed of  $80 \text{ km}\cdot\text{h}^{-1}$ . A car M going along the second road is 600 m from the crossroad. What is the speed (in  $\text{km}\cdot\text{h}^{-1}$  and  $\text{m}\cdot\text{s}^{-1}$ ) of the car M if the two cars crash at the crossroad? Sketch the v-t and s-t graphs of both cars. Use a pencil and ruler.
13. Two bodies are moving at the same time from two points M and N lying on the x-axis in the same direction along this axis. These points are 100 m from each other. The body moving from M has a speed of  $5 \text{ m}\cdot\text{s}^{-1}$ , the second one moving from N has a speed of  $3 \text{ m}\cdot\text{s}^{-1}$ . When does the first body catch the second one? What are the distances travelled by both bodies? Sketch the v-t and s-t graphs of both cars. Use a pencil and ruler.

14. A pedestrian moving with uniform velocity covers a distance of 9 m in the first 6 seconds, in the following 4 seconds he covers a distance of 8 m. What is his speed in the first 6 and the following 4 seconds? What is his average speed in the first 10 seconds (in  $\text{km}\cdot\text{h}^{-1}$  and  $\text{m}\cdot\text{s}^{-1}$ )? Sketch the v-t and s-t graphs of his real movement. Use a pencil and ruler.

15. A car moves in the first half of the distance travelled at  $30 \text{ km}\cdot\text{h}^{-1}$  and the second half at  $50 \text{ km}\cdot\text{h}^{-1}$ . A second car which starts at the same time moves along the same trajectory at a uniform speed of  $40 \text{ km}\cdot\text{h}^{-1}$ . Which of them will reach the destination first?

### Adding velocities

The resultant velocity of the body can have more components – see the examples below.

#### Questions:

16. A ship floats on a lake; its speed relative to the water is  $3 \text{ km}\cdot\text{h}^{-1}$ . A passenger A walks on the deck in the direction of motion of the ship at  $3 \text{ km}\cdot\text{h}^{-1}$ , passenger B walks on the deck against the direction of motion of the ship at  $3 \text{ km}\cdot\text{h}^{-1}$ , passenger C just stays on the deck on the same place. What is the speed of each passenger relative to the water in the lake?

17. Water in the river flows at  $2 \text{ m}\cdot\text{s}^{-1}$ . A motor boat can produce a speed of  $12 \text{ m}\cdot\text{s}^{-1}$ .

- What should be the angle at which the motor boat moves relative to the banks so that its final direction of motion is perpendicular to the banks of the river?
- How long will it take to cross the river assuming the banks are 800 m apart?

18. A boat is being rowed across a river 0.7 km wide which is flowing at  $0.8 \text{ m}\cdot\text{s}^{-1}$  and its front is perpendicular to the banks. If the speed of the boat relative to the water is  $0.6 \text{ m}\cdot\text{s}^{-1}$  calculate

- the velocity of the boat relative to the banks
- the time taken to cross the river
- the lateral displacement when the boat reaches the far bank

19. Rain is falling vertically at  $8 \text{ m}\cdot\text{s}^{-1}$  relative to the ground. The raindrops make tracks on the side window of a car at  $30^\circ$  relative to the horizontal. What is the speed of the car?

20. A boy is swimming steadily at  $0.85 \text{ m}\cdot\text{s}^{-1}$  relative to water. The speed of the current in the river is  $0.4 \text{ m}\cdot\text{s}^{-1}$  and the width of the river is 60 m. Calculate

- the resultant speed of the swimmer relative to the bank of the river if he is finally moving perpendicularly to the banks
- the time taken to cross the river.

21. A ball is thrown at an initial speed of  $7 \text{ m}\cdot\text{s}^{-1}$  in the carriage of a fast train travelling at a steady speed of  $24 \text{ m}\cdot\text{s}^{-1}$ . Calculate the initial speed of the ball relative to the Earth if we

- throw it in the direction of the motion
- in the opposite direction
- perpendicularly to the direction of the movement of the train

L2/6-8, 11, 16, 19, 25, 32, 34

### 3. Steadily accelerated and steadily decelerated linear motion

During the same time intervals there is the same change in speed. When we take the time interval equal to one second, the change in speed represents the acceleration of the motion.

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \text{const}$$

$$[a] = \frac{[\Delta v]}{[\Delta t]} = \frac{\text{m} \cdot \text{s}^{-1}}{\text{s}} = \text{m} \cdot \text{s}^{-2}$$

- equations for distance and speed (*will be derived from the following examples*)

$$s = s_0 + v_0 t \pm \frac{1}{2} a t^2$$

$$v = v_0 \pm a t$$

[http://phet.colorado.edu/simulations/sims.php?sim=The\\_Moving\\_Man](http://phet.colorado.edu/simulations/sims.php?sim=The_Moving_Man)  
<http://www.walter-fendt.de/ph14e/acceleration.htm>

### without initial speed

#### Questions:

22. A locomotive accelerates steadily from rest and it reaches the speed of  $10 \text{ m} \cdot \text{s}^{-1}$  in ten seconds.
- Calculate the acceleration
  - Sketch v-t and a-t graphs
  - Use the v-t graph to state the distance travelled
  - Draw the s-t graph
23. A car moves from rest and it reaches the speed of  $90 \text{ km} \cdot \text{h}^{-1}$  in 20 seconds. Assume steadily accelerated motion and calculate acceleration and the distance travelled during this time. Sketch v-t graph.

L2/37, 38

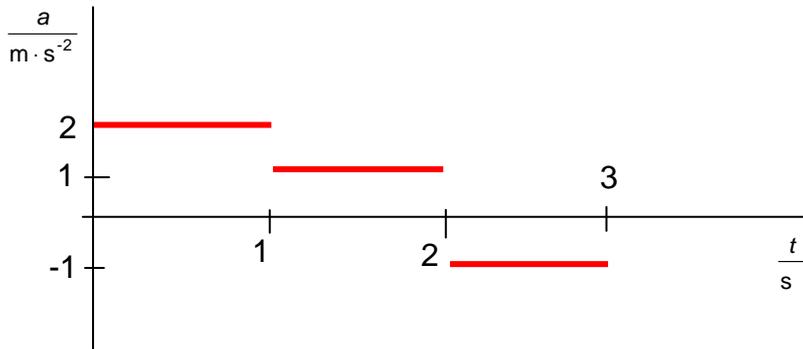
### with initial speed

#### Questions:

24. The initial speed of a mass point is  $10 \text{ m} \cdot \text{s}^{-1}$  and it provides a linear motion. Assume its acceleration to be  $3 \text{ m} \cdot \text{s}^{-2}$  and calculate the speed of the point 5 s after the beginning of the measurement. Sketch v-t graph and try to calculate the distance travelled during this time interval.
25. The speed of a train has changed during 40 s from  $36 \text{ km} \cdot \text{h}^{-1}$  to  $18 \text{ km} \cdot \text{h}^{-1}$ . Calculate the acceleration and distance travelled. Prove using the v-t graph.
26. A car initially moving at  $80 \text{ km} \cdot \text{h}^{-1}$  has raised its speed regularly during 10 seconds to  $100 \text{ km} \cdot \text{h}^{-1}$ . State the acceleration and distance travelled.
27. A car moving at  $50 \text{ km} \cdot \text{h}^{-1}$  has to stop in 12.5 m. Calculate its acceleration and the time taken to stop.
28. A train lowers its speed steadily from  $80 \text{ km} \cdot \text{h}^{-1}$  to  $60 \text{ km} \cdot \text{h}^{-1}$  during 40 seconds. Calculate the acceleration and distance travelled.
29. In the electric field of a cathode ray tube an electron is steadily accelerated from  $5 \text{ km} \cdot \text{s}^{-1}$  to  $20\,000 \text{ km} \cdot \text{s}^{-1}$  over a distance of 10 mm. What is its acceleration?
30. A body is moving from rest with acceleration  $8 \text{ m} \cdot \text{s}^{-2}$ . Calculate the velocity at the end of the distance of length 100 m.

31. A cyclist riding at  $3 \text{ m}\cdot\text{s}^{-1}$  starts to move with acceleration and reaches the speed of  $7 \text{ m}\cdot\text{s}^{-1}$  in 8 s. Assume steadily accelerated motion and stipulate the magnitude of acceleration and distance travelled during the accelerated motion.

32. Draw v-t graph related to the given a-t graph. Assume zero initial speed.



33. Two cars move from the same point. The first one already has the speed of  $2 \text{ m}\cdot\text{s}^{-1}$  and it is steadily accelerated by  $0.5 \text{ m}\cdot\text{s}^{-2}$ . The second one was at rest and it has acceleration  $1 \text{ m}\cdot\text{s}^{-2}$ , but it starts to move 5 seconds after the first one. Calculate a) when the cars will have the same speed and what the value of this speed is b) when and how far from the starting point the cars will meet c) sketch v-t and s-t graphs.

34. Two motorcycles are 4 km apart and they move towards each other. The first one has initial speed  $2 \text{ m}\cdot\text{s}^{-1}$  and acceleration  $0.25 \text{ m}\cdot\text{s}^{-2}$ , the second one has initial speed  $4 \text{ m}\cdot\text{s}^{-1}$  and acceleration  $0.5 \text{ m}\cdot\text{s}^{-2}$ . Calculate when and how far from the starting point of the first motorcycle they will collide.

35. Two objects move from the same point at the same time. The first one has initial speed  $6 \text{ m}\cdot\text{s}^{-1}$  and acceleration  $0.2 \text{ m}\cdot\text{s}^{-2}$ , the second one has initial speed  $2 \text{ m}\cdot\text{s}^{-1}$  and acceleration  $3 \text{ m}\cdot\text{s}^{-2}$ . Calculate a) when the objects will have the same speed and what the value of this speed is b) when and how far from the starting point the objects will meet c) sketch v-t and s-t graphs.

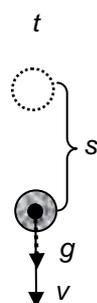
L2/ 40-52, 55-57

#### 4. Free fall

- in a VACUUM!!!
- in the gravitational field NEAR the Earth's surface  
 $a = g = 9.81 \text{ m}\cdot\text{s}^{-2} = 10 \text{ m}\cdot\text{s}^{-2}$
- from rest ( $v_0 = 0$ )



$v_0 = 0$



$$\text{distance fallen: } s = s_0 + v_0 t + \frac{1}{2} a t^2 = \frac{1}{2} g t^2$$

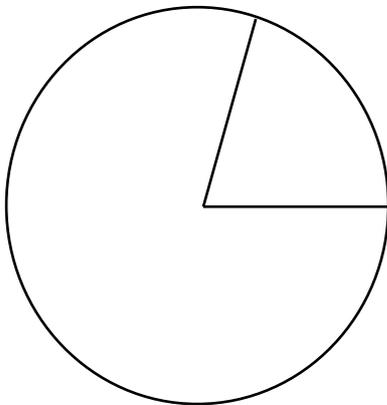
$$\text{speed of falling object: } v = v_0 \pm a t = g t$$

<http://www.walter-fendt.de/ph14e/projectile.htm>

**Questions:**

36. What is the time taken for the stone to fall 80 m? Does it correspond with reality? Explain.
37. A car hits an obstacle when moving at  $60 \text{ km}\cdot\text{h}^{-1}$ . What should be the height needed for the car to fall freely to get the same speed?
38. State the time interval during which the speed of an object falling freely rises from  $10 \text{ m}\cdot\text{s}^{-1}$  to  $30 \text{ m}\cdot\text{s}^{-1}$ .
39. What is the distance fallen by the object during the fifth second of the fall?

**5. Steady circular motion**



**angle in radians**

$$\varphi = \frac{\text{length of an arc}}{\text{radius}} = \frac{s}{r}$$

full angle in radians

$$\varphi = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

$$\Rightarrow 360^\circ = 2\pi \text{ rad}$$

**angular velocity**  $\omega = \frac{\varphi}{t}$   $[\omega] = \text{rad} \cdot \text{s}^{-1}$

- S.c.m. is one of the basic types of **periodic motion** – trajectory (in this case a circle) is repeated.

**period T** ... time taken to complete one cycle (=circle)

**frequency f** ... number of cycles (circles) completed in one second

$$f = \frac{1}{T} \quad [f] = \text{s}^{-1} = \text{Hz (hertz)}$$

- links between**  $\omega, v, T, f$

$$\omega = \frac{\varphi}{t} = \frac{s}{r \cdot t} = \frac{v}{r}$$

$$\omega = \frac{\text{any angle}}{\text{time taken}} = \frac{\text{full angle}}{\text{period}} = \frac{2\pi}{T} = 2\pi f$$

choose

[http://phet.colorado.edu/simulations/sims.php?sim=Ladybug\\_Revolution](http://phet.colorado.edu/simulations/sims.php?sim=Ladybug_Revolution)

**Questions:**

40. A record player makes 33 revolutions per minute. Calculate
- period and frequency of the motion
  - angular velocity and speed of the point 12 cm from the centre of rotation

41. A bob provides a s.c.m. The radius of the circle is 0.5 m and an arc of length 40 cm was travelled in 0.1 seconds. Calculate the period and frequency of the motion, angular velocity and speed of the bob.
42. A bicycle wheel has the diameter of 0.5 m and it makes 7 rotations per second. Find the angular velocity and the speed of a point on the tyre.
43. Stipulate angular velocity of the shaft which performs 120 rotations in 1 min. Find the period and frequency of this motion.
44. What is the speed of points on the Earth's equator? The radius of the Earth is 6 378 km. The angular velocity is  $7.29 \times 10^{-5} \text{ rad} \cdot \text{s}^{-1}$ . Stipulate also the frequency.
45. A pulsar of diameter 15 km rotates at a frequency of 9 Hz. Calculate the speed of a point on the equatorial diameter of the pulsar.
46. A vehicle has tyres of diameter 0.55 m. Work out the angular speed of a point on the tyre tread when the car is travelling at  $30 \text{ m} \cdot \text{s}^{-1}$ .
47. An artificial satellite is moving in a circular orbit of radius nearly 42 250 km. Calculate its linear velocity, if it takes 24 hour to revolve round the Earth.
48. Find appropriate angles in radians for  $45^\circ$ ,  $30^\circ$ ,  $60^\circ$ ,  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ ,  $50^\circ$ ,  $75^\circ$
49. Find appropriate angles in degrees for 1 rad,  $\pi$  rad,  $\pi/2$  rad,  $\pi/3$  rad,  $\pi/4$  rad,  $\pi/6$  rad,  $\pi/8$  rad

L2/68-74a

**Answers:**

- |                                                                                                                   |                                                                                                   |
|-------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| 1. 20 km                                                                                                          | 26. $0.56 \text{ m} \cdot \text{s}^{-2}$ , 250 m                                                  |
| 2. 200 m                                                                                                          | 27. $-7.7 \text{ m} \cdot \text{s}^{-2}$ , 1.8 s                                                  |
| 4. $59.4 \text{ km} \cdot \text{h}^{-1}$                                                                          | 28. $-0.14 \text{ m} \cdot \text{s}^{-2}$ , 778 m                                                 |
| 5. $58.5 \text{ km} \cdot \text{h}^{-1}$                                                                          | 29. $2 \times 10^{16} \text{ m} \cdot \text{s}^{-2}$                                              |
| 6. $57.6 \text{ km} \cdot \text{h}^{-1}$                                                                          | 30. $40 \text{ m} \cdot \text{s}^{-1}$                                                            |
| 7. $5.3 \text{ m} \cdot \text{s}^{-1}$                                                                            | 31. $0.5 \text{ m} \cdot \text{s}^{-2}$ , 40 m                                                    |
| 8. $68.75 \text{ km} \cdot \text{h}^{-1}$                                                                         | 33. a) 14 s, $9 \text{ m} \cdot \text{s}^{-1}$ b) 26 s, 221 m                                     |
| 9. $5 \text{ km} \cdot \text{h}^{-1}$                                                                             | 34. 95.6 s, 1 333 m                                                                               |
| 10. $78.26 \text{ km} \cdot \text{h}^{-1}$                                                                        | 35. a) 1.43 s, $6.3 \text{ m} \cdot \text{s}^{-1}$ b) 2.86 s, 18 m                                |
| 11. I. $5 \text{ m} \cdot \text{s}^{-1}$ , $8.3 \text{ m} \cdot \text{s}^{-1}$                                    | 36. 4 s                                                                                           |
| II. $50 \text{ km} \cdot \text{h}^{-1}$ , $33.3 \text{ km} \cdot \text{h}^{-1}$                                   | 37. 13.9 m                                                                                        |
| 12. 0.01 h, $60 \text{ km} \cdot \text{h}^{-1}$                                                                   | 38. 2 s                                                                                           |
| 13. 250 m, 150 m, 50 s                                                                                            | 39. 45 m                                                                                          |
| 14. $1.5 \text{ m} \cdot \text{s}^{-1}$ , $2 \text{ m} \cdot \text{s}^{-1}$ , $1.7 \text{ m} \cdot \text{s}^{-1}$ | 40. a) 0.55 Hz, 1.818 s                                                                           |
| 15. the second one                                                                                                | b) $3.46 \text{ rad} \cdot \text{s}^{-1}$ , $0.41 \text{ m} \cdot \text{s}^{-1}$                  |
| 16. A $6 \text{ km} \cdot \text{h}^{-1}$ , B 0, C $3 \text{ km} \cdot \text{h}^{-1}$                              | 41. 0.79 s, 1.27 Hz, $8 \text{ rad} \cdot \text{s}^{-1}$ , $4 \text{ m} \cdot \text{s}^{-1}$      |
| 17. $11.8 \text{ m} \cdot \text{s}^{-1}$ , $80.4^\circ$ , 68 s                                                    | 42. $44 \text{ rad} \cdot \text{s}^{-1}$ , $11 \text{ m} \cdot \text{s}^{-1}$                     |
| 18. $1 \text{ m} \cdot \text{s}^{-1}$ at $53^\circ$ to heading, 1 167 s, 933 m                                    | 43. 2 Hz, 0.5 s, $12.6 \text{ rad} \cdot \text{s}^{-1}$                                           |
| 19. $13.9 \text{ m} \cdot \text{s}^{-1}$                                                                          | 44. $465 \text{ m} \cdot \text{s}^{-1}$ , $1.16 \times 10^{-5} \text{ Hz}$                        |
| 20. $0.75 \text{ m} \cdot \text{s}^{-1}$ , 80 s                                                                   | 45. $424 \text{ km} \cdot \text{s}^{-1}$                                                          |
| 21. $31 \text{ m} \cdot \text{s}^{-1}$ , $17 \text{ m} \cdot \text{s}^{-1}$ , $25 \text{ m} \cdot \text{s}^{-1}$  | 46. $109 \text{ rad} \cdot \text{s}^{-1}$                                                         |
| 22. $1 \text{ m} \cdot \text{s}^{-1}$ , 50 m                                                                      | 47. $3 \text{ km} \cdot \text{s}^{-1}$                                                            |
| 23. $1.25 \text{ m} \cdot \text{s}^{-2}$ , 250 m                                                                  | 48. $\pi/4$ rad, $\pi/6$ rad, $\pi/3$ rad, $\pi/2$ rad, $\pi$ rad,                                |
| 24. $25 \text{ m} \cdot \text{s}^{-1}$ , 87.5 m                                                                   | $3\pi/2$ rad, 0.87 rad, 1.31 rad                                                                  |
| 25. $-0.125 \text{ m} \cdot \text{s}^{-2}$ , 300 m                                                                | 49. $57.3^\circ$ , $180^\circ$ , $90^\circ$ , $60^\circ$ , $45^\circ$ , $30^\circ$ , $22.5^\circ$ |