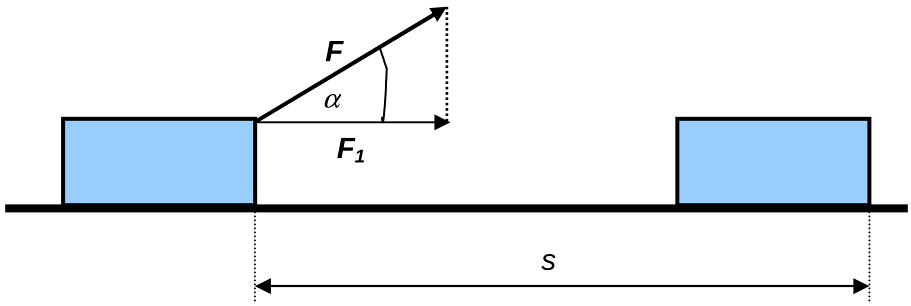
MECHANICAL WORK AND ENERGY

1. Work (*W*)

In this topic we are going to discuss mechanical work only – an object is moved because of force acting on it.

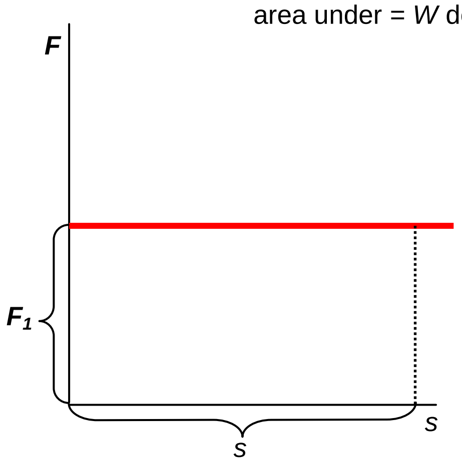


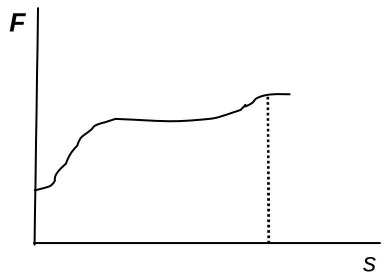
Mechanical work is done only when a force acting on it has a component in or against the direction of motion.

$$W = F_1 s = Fs\cos\alpha$$
$$[W] = \mathrm{J}$$

force-displacement graph

area under = *W* done (for ANY shape!)





2. Power (*P*)

is the rate of change of work

a) **average power** – the work was finished during certain time interval *t*

$$P = \frac{W}{t}$$
$$[P] = \mathrm{J}\cdot\mathrm{s}^{-1} = \mathrm{W}\ (\mathrm{watt})$$

b) **instantaneous power** – when the work is still being done (object moves)

$$P = \frac{Fs}{t} = Fv$$

F... force applied by a machine/man
v ... speed of a moving object (needn't be constant $v = v_0 + at$)

3. Efficiency (η)

$$\eta = \frac{\text{energy/work/power OUTPUT}}{\text{energy/work/power INPUT}} = \frac{E}{E_0} = \frac{W}{W_0} = \frac{P}{P_0} \leq 1 \text{ or } 100\%$$

Questions:
1. A resultant force of 80 N is applied on a stationary body for 5 seconds. Assume the mass of the body 2 kg and calculate:
a) the total work done by the resultant
b) its average power
c) instantaneous power 5 seconds from the beginning of the motion
d) compare b) and c), discuss

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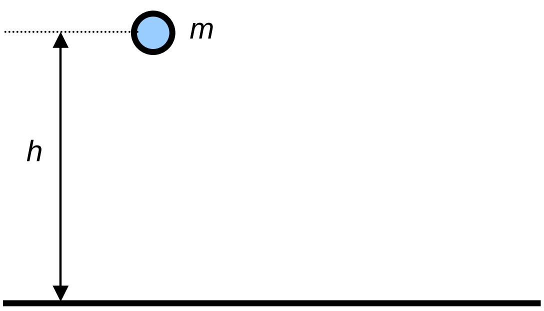
4. Mechanical energy (E, E_{mech})

Mechanical energy of an object is defined as its ability to do mechanical work.

a) **potential** (E_p) – has a system of bodies due to their relative position

elastic energy – has a deformed object – stretched or compressed spring, ...

in the gravitational field – has an object at certain height related to the ground or a chosen „zero level of E_p"

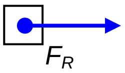


$E_p = W = Fs = mgh$

$E_p = 0$

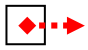
b) **kinetic** (E_k)

$t = 0,\ v_0 = 0$ $\qquad$ $t,\ v = at,\ E_k$



F_R

resultant force acts
during time t



v

work done by the resultant = change in kinetic energy

$$W = F_R s = ma\frac{1}{2}at^2 = \frac{1}{2}ma^2t^2 = \frac{1}{2}mv^2 = E_k$$

The law of conservation of energy in an isolated system = the total amount of energy in an isolated system remains constant. This includes heat as well.

Mechanical energy is conserved only when not converted into heat e.g. because of friction etc.= **the law of conservation of mechanical energy in an isolated system.**

In some examples we can calculate how much energy was converted into heat or in fact internal energy using a simple equation:

$$E_{mi} = E_{mf} + \Delta U$$

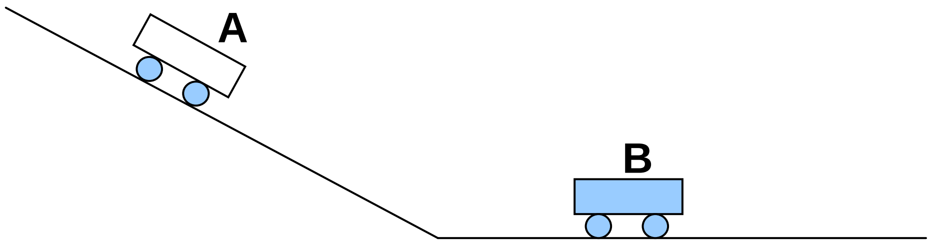
E_{mi} ... initial mechanical energy
E_{mf} ... final mechanical energy
ΔU ... change in internal energy, energy converted into other types

http://phet.colorado.edu/simulations/sims.php?sim=Energy_Skate_Park

Questions:

2. An elastic ball of mass 50 g is released from rest at a height of 1.55 m above a rigid horizontal metal plate. After the rebound the ball rises vertically to a height of 1.00 m above the plate. Calculate
a) the velocity of the ball just before impact
b) the momentum of the ball just before impact
c) the kinetic energy of the ball just before impact
d) the loss of energy on impact. Give reasons. (g = 10 m·s^{-2})

3. Figure below shows a trolley A of mass 4 kg with light frictionless wheels which is held at rest on a smooth inclined plane. When it is released its centre of mass is lowered through a vertical distance of 3.45 m while it is accelerating down the slope. It then collides with the second similar trolley B of mass 2.5 kg which is at rest on a horizontal plane. After the collision the two trolleys travel forward as a single body.



a) Calculate the speed of A immediately before the collision.
b) Use the principle of conservation of momentum to find the common speed of the two trolleys after the collision.
c) Calculate the kinetic energy of A just before the collision and the kinetic energy of the two trolleys after the collision.
d) Explain why the kinetic energies before and after the collision are different. (g = 10 m·s^{-2})

4. A trolley A of mass 2 kg is moving at 2 m·s^{-1} and a trolley B of mass 5 kg is moving in the opposite direction at 4 m·s^{-1}.
a) Assuming an inelastic collision, how much of their kinetic energy is converted into other forms of energy?
b) If the collision had been elastic, what would the velocities of the trolleys have been after separation?

5. A ball of mass 0.25 kg is dropped from a height of 2.0 m onto a flat surface and rebounds to a height of 0.38 m. Disregard air resistance and calculate
a) the speed of the ball just before impact
b) the speed of the ball just after impact
c) the change of momentum of the ball due to impact
d) the force of impact, if the time of contact at the surface was 0.070 s.

6. A railway wagon of mass 1800 kg travelling at a speed of 1.7 m·s^{-1} collides with three identical wagons initially at rest. The wagons couple together as a result of the impact. Calculate
a) the speed of the wagons after impact
b) the loss of kinetic energy due to the impact
c) assume that the three wagons were initially moving at 1 m·s^{-1} in the opposite direction to the single wagon and calculate the speed of the wagons after impact and the loss of kinetic energy due to the impact

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Answers:
1. 40 kJ, 8 kW, 16 kW
2. 5.57 m·s^{-1}, 0.28 kg·m·s^{-1}, 0.775 J, 0.275 J
3. 8.3 m·s^{-1}, 5.1 m·s^{-1}, 138 J, 84.5 J
4. 26 J, -46/7 m·s^{-1}, -4/7 m·s^{-1}
5. 6.3 m·s^{-1}, 2.8 m·s^{-1}, 2.28 kg·m·s^{-1}, 32.5 N
6. 0.425 m·s^{-1}, 1951 J, -0.325 m·s^{-1}, 4921 J