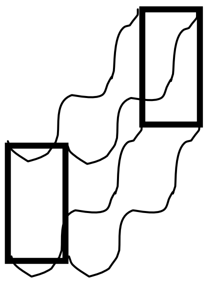
MECHANICS OF SOLIDS

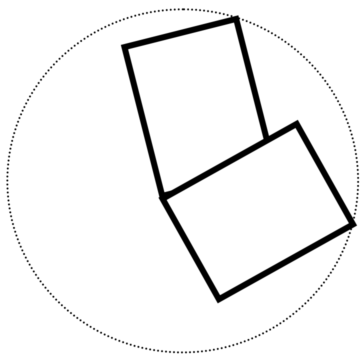
1. Basic ideas

- **solid (rigid) body** – ideal object which cannot be deformed using any value of force
- **types of motion of a rigid body**

  a) translation – all the points of the object describe the same trajectory
  b) rotation – all the points of the object move in circles which MUST have their centres on the same axis = axis of rotation
  c) combined motion – from the previous ones

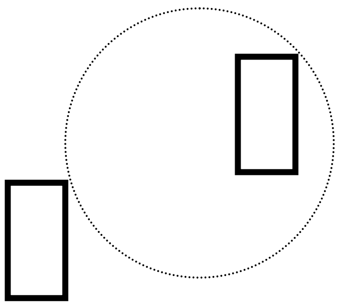


a)



b)

Which type of motion is this?

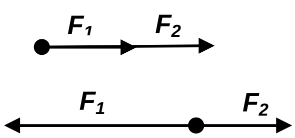


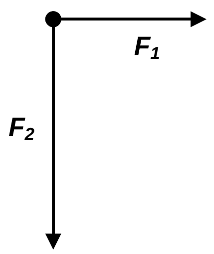
2. Adding forces

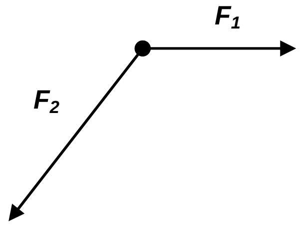
means to replace two (or more) forces by their resultant, which must have the same effect as all the forces together

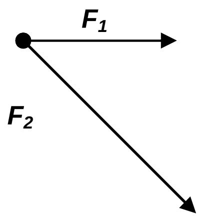
a) forces have the same point of action ≈ as if they act on a mass point

the resultant graphically | its size | its direction



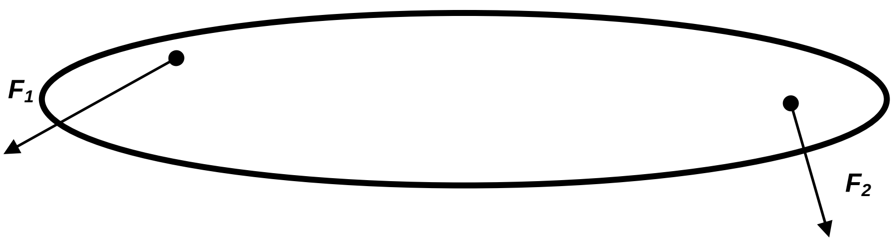


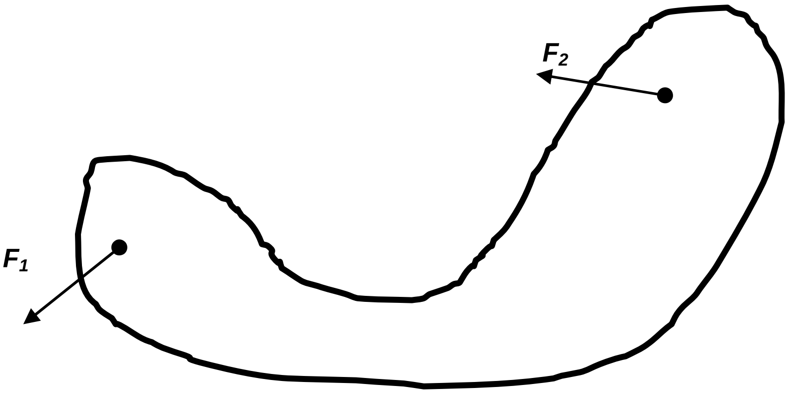




b) forces have different point of action

i) diverging lines of action

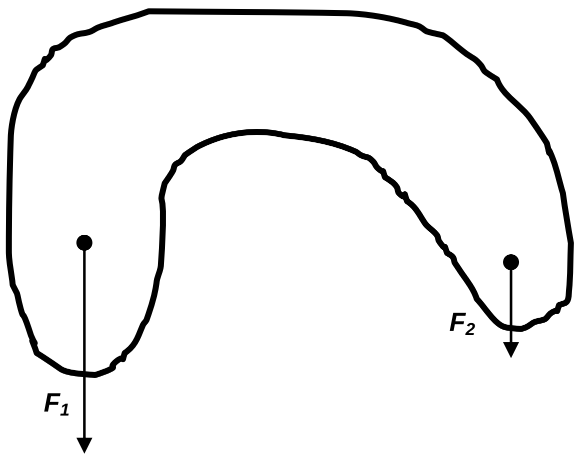




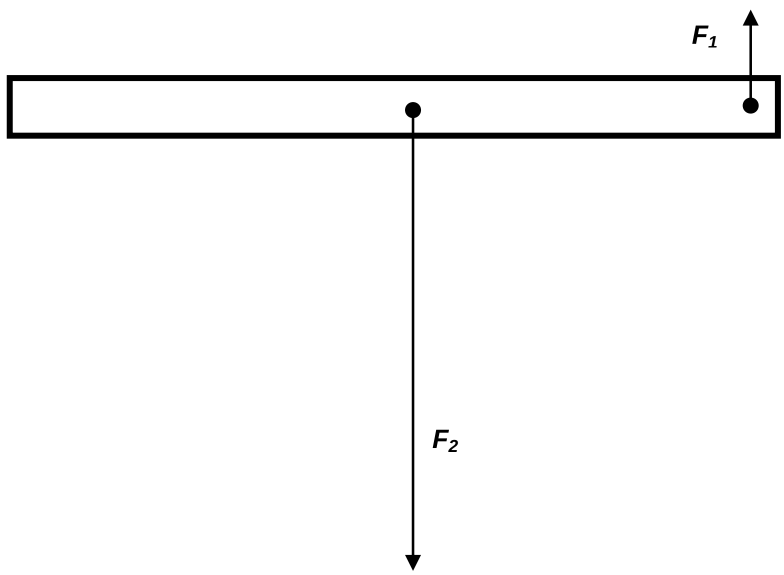
When the line of action of the resultant does not cross the object, the forces cannot be replaced by the resultant.

ii) parallel lines of action

the same direction
– always possible to find the resultant
(the resultant is placed „between“)



opposite direction
- when the line of action of the resultant does not cross the object, the forces cannot be replaced by the resultant (the resultant is placed „out“ closer to the bigger force)



size and direction of the resultant: as if the forces act from one point

point of action of the resultant – problem, calculation later – now:

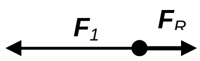
<u>*graphical way:*</u>

- swap the forces and change the direction of ONE $\Rightarrow F_1^{/}, F_2^{/}$
- *point of intersection* of the line joining the points of action of F_1 and F_2 and the line joining the tips of $F_1^{/}, F_2^{/}$ = *point of action of the resultant*

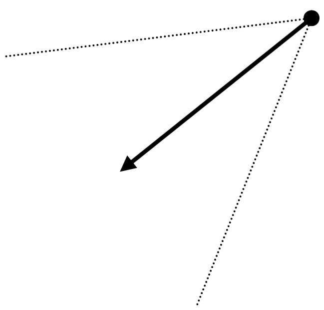
3. Resolving forces

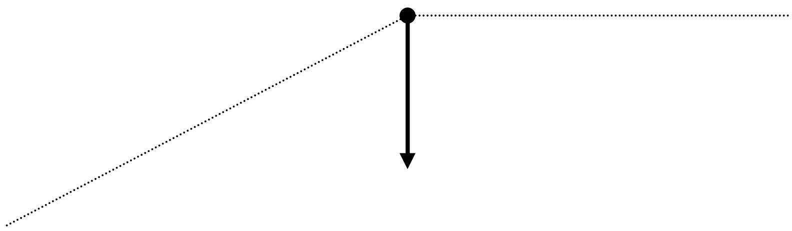
means to find one or both components of the resultant (used e.g. for objects on inclined planes to find the normal component of gravity important for friction and the kinetic component parallel to the plane)

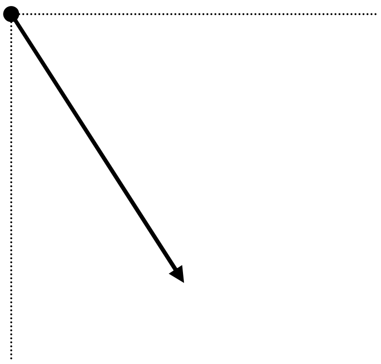
find $\boldsymbol{F_2}$

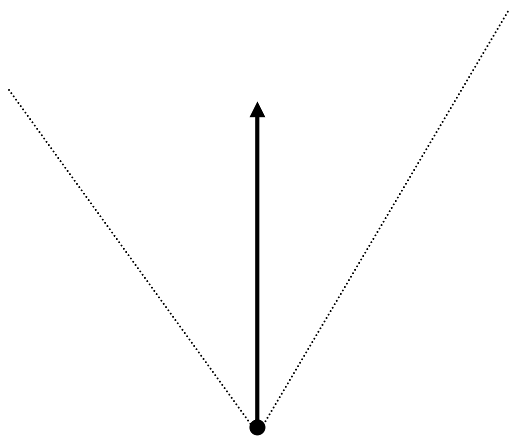


find $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ in the given directions



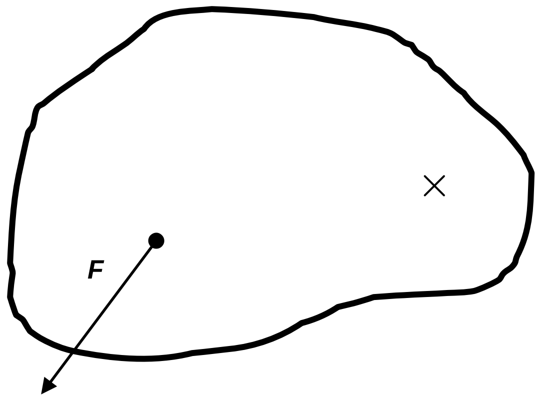






4. Moment of a force ($\vec{M}$)

is a vector quantity used to describe the turning effect of a force.
size : $M = F\,r$



direction : right-hand-grip rule
grip the axis fingers in the direction of rotation, thumb shows the direction of the moment (from or into/behind)

$[M] = \mathrm{N \cdot m}$ *(newtonmeter, not joule !!!)*

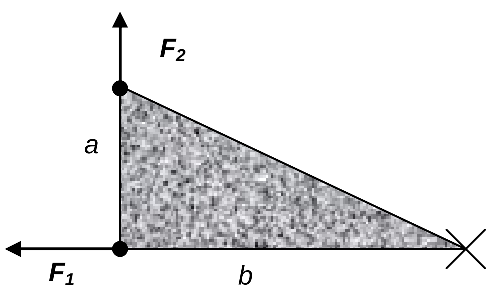
5. Resultant moment of more forces acting on one object

2 methods, each suitable for different situations:

- **add the forces and then state the moment of the resultant**
  $\vec{F_1} + \vec{F_2} + ... + \vec{F_n} = \vec{F_R}$
  size : $M = F_R\, r$
- **state the moments of all the forces and add them <u>as vectors</u>**
  $\vec{M_1} + \vec{M_2} + ... + \vec{M_n} = \vec{M_R}$

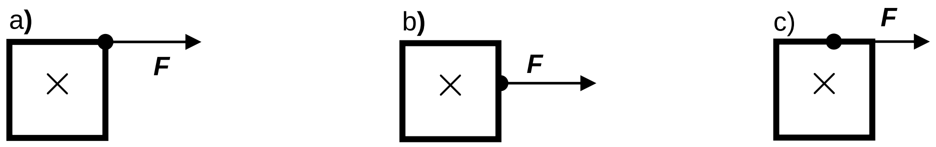
L2/ 261-3, 265-6, x267-8, 269-270, 273-275, x283-4

Questions:
1.
$F_1 = 5\,\mathrm{N}$
$F_2 = 4\,\mathrm{N}$
$a = 5\,\mathrm{cm}$
$b = 7\,\mathrm{cm}$

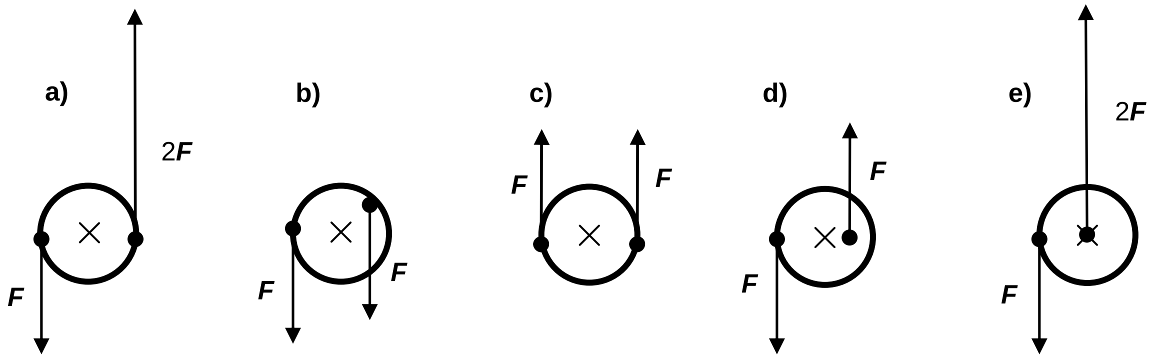


Calculate the resultant force and moment and state its direction.

2. $F = 4$ N, $a = 20$ cm, $\vec{M} =$?

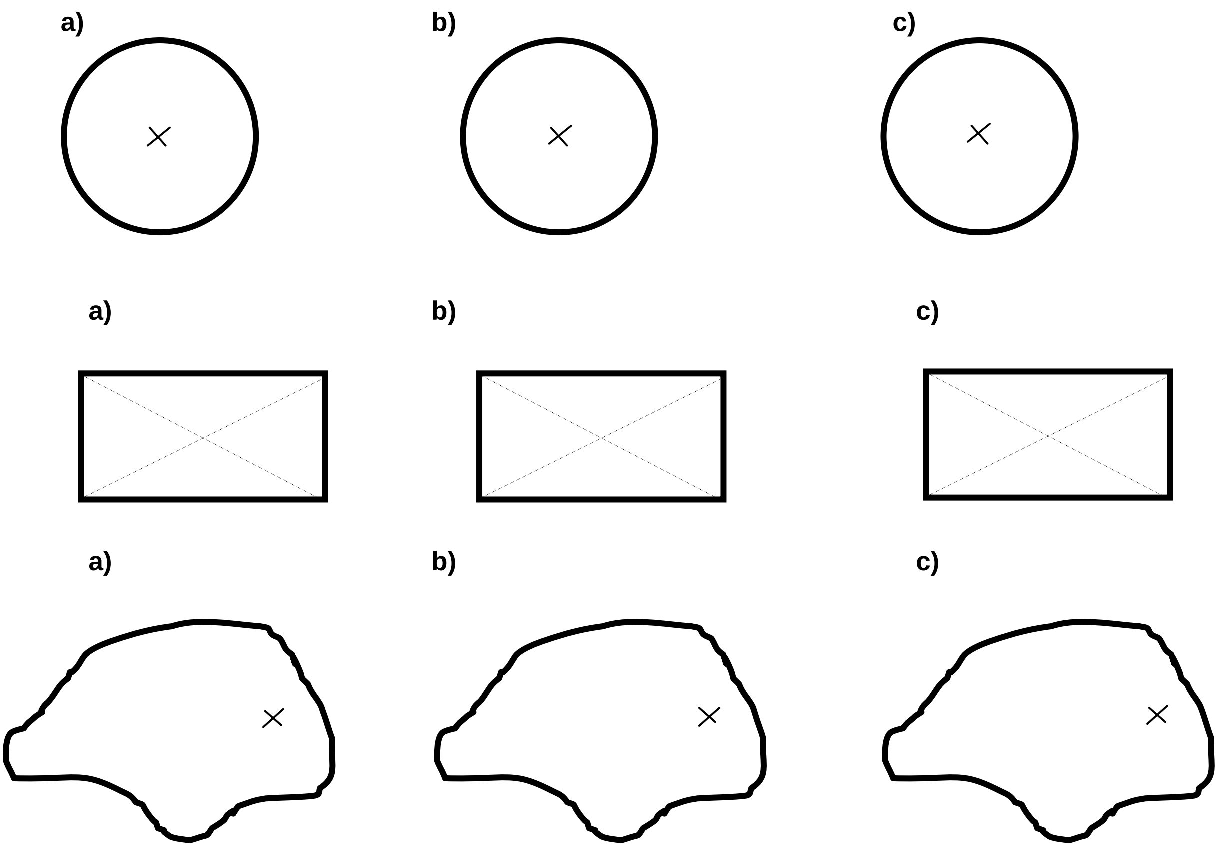


3. Which of the following examples represents a couple? Sort the figures according to the rising moment.



4. Draw a force of size 5 N, which has

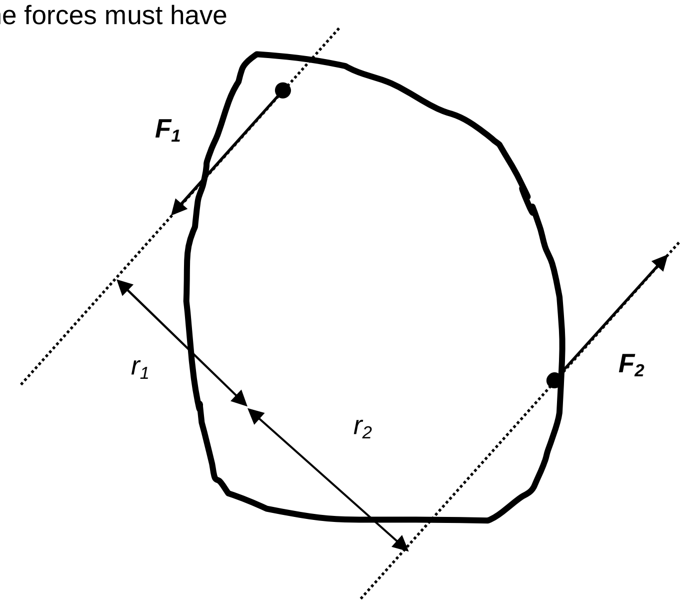
a) no turning effect
b) maximum turning effect
c) a couple with maximum turning effect



6. Couple

A couple consists of two special forces acting on an object; the forces must have

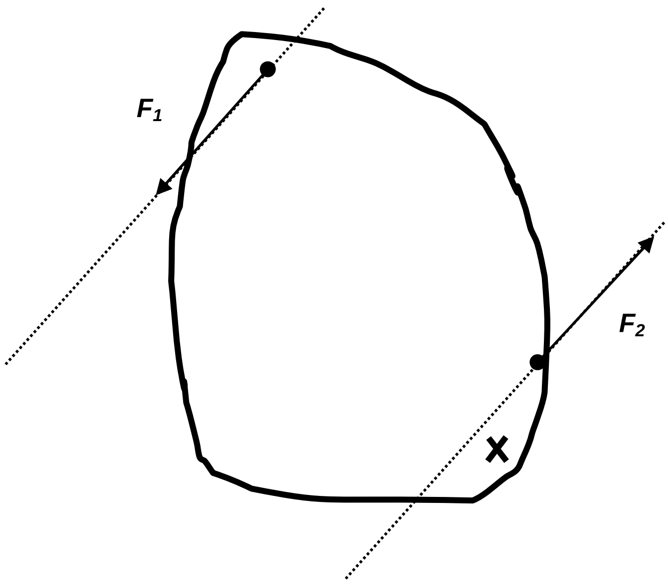
- the same size $F_1 = F_2$
- opposite direction
- parallel lines of action



size of $\boldsymbol{M_R} = D$

$$D = M_1 + M_2 = F_1 r_1 + F_2 r_2 =$$
$$= F_1 (r_1 + r_2) = F_1 d$$

Derive the equation for D for the new position of the axis:



$\Rightarrow$ **the couple of forces**

- cannot be replaced by the resultant
- its moment does NOT depend on the position of the axis (hinge)

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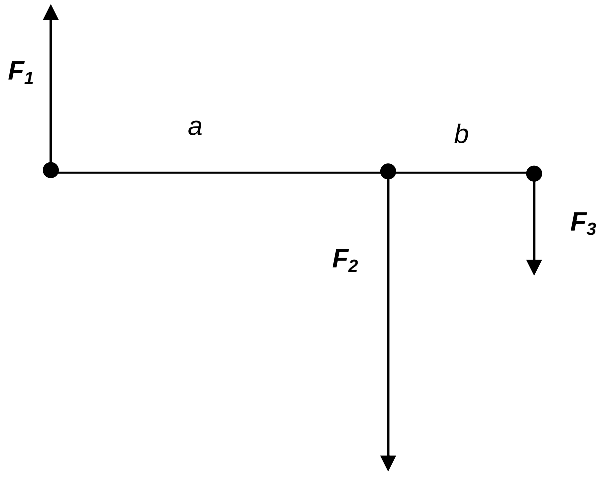
L2/273 Find the resultant force of the three forces in the figure

a = 0.6 m
b = 0.3 m
F_1 = 50 N
F_2 = 80 N
F_3 = 30 N



Similar situations can be solved using the idea of turning effect of forces, though there is in fact no axis and no rotation!!! If we replace the forces by their resultant, nothing can change. So the turning effect, i.e. moment of the resultant force related to ANY axis of rotation, must be the same as the sum of moments (as vectors!) of the particular forces.

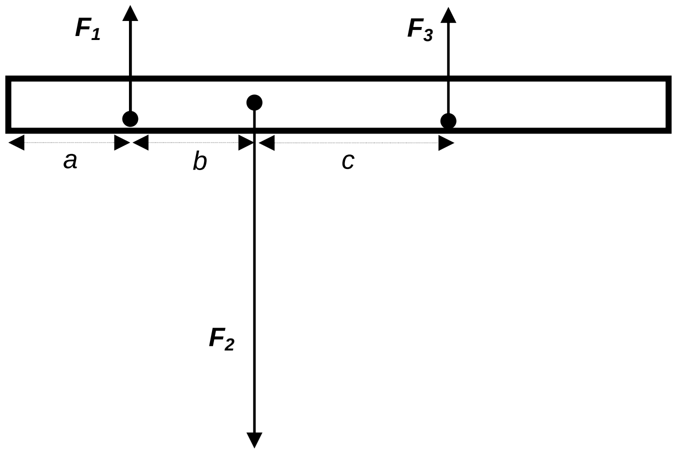
This method in practice:

- choose the axis of rotation
  - can be at any point, the advantage is to choose the point of action of one of the forces so that its moment is equal to zero
- calculate the size of the resultant
- choose the positive direction from the axis and the positive direction of moments (from or into)
- write the equation: moment of the resultant = sum of moments of the forces about the chosen axis
  - do not forget to take the moments as + or – (when the force would rotate the object in the opposite way than the resultant)
- write the result in a clear way what the distance is from the left/right side of the object in proper units, or make a simple sketch
- as a control – choose another axis; the point of action of the resultant must be at the same point in reality, though „*d*" will differ.

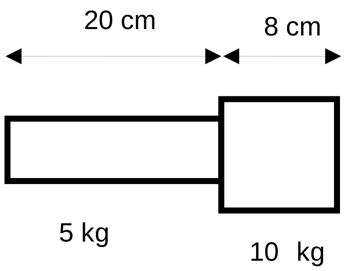
Questions:

5. Find the size and the point of action of the resultant from the LEFT side of the object.

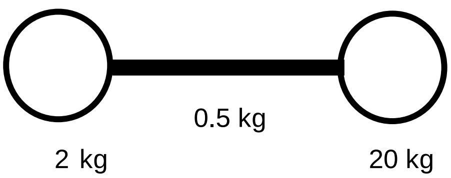
l = 2 m (total length of the object)
a = 0.4 m
b = 0.4 m
c = 0.6 m
F_1 = 10 N
F_2 = 30 N
F_3 = 10 N



6. Find the size and the point of action of the resultant from the RIGHT side of the object.
= the centre of mass of the object



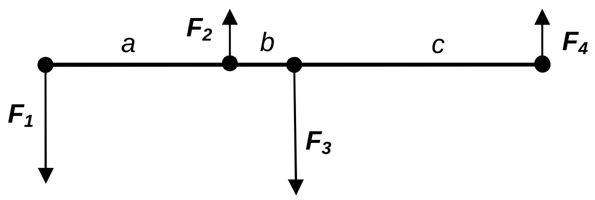
7. Find the centre of mass of the object.



diameter of the spheres is 20 cm
length of the rod joining them is 40 cm

8. Find the magnitude of the resultant force and the point of action for the system of four parallel forces.

F_1 = 40 N a = 0.7 m
F_2 = 25 N b = 0.2 m
F_3 = 45 N c = 0.9 m
F_4 = 25 N



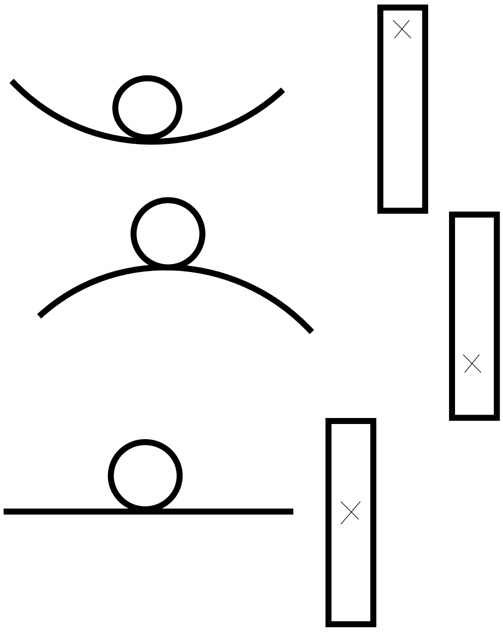
7. Equilibrium and stability

- **Equilibrium conditions**: both the resultant force and the resultant moment must be equal to zero.
- The measure of stability of an object is the work done to move/turn it from stable to unstable equilibrium
- **Types of equilibria**

a) stable

b) unstable

c) neutral



Questions:
9. What work must be done to turn a prism around its edge from the stable to unstable equilibrium? The prism stands on the square face of a = 30 cm, the height of the prism is h = 40 cm and the density of the prism is ρ = 2 500 kg·m^{-3}. Assume g = 10 m·s^{-2}.

10. State if it is more efficient to turn the cube or to push it, assume the coefficient of friction 0.3.

L2/279-282

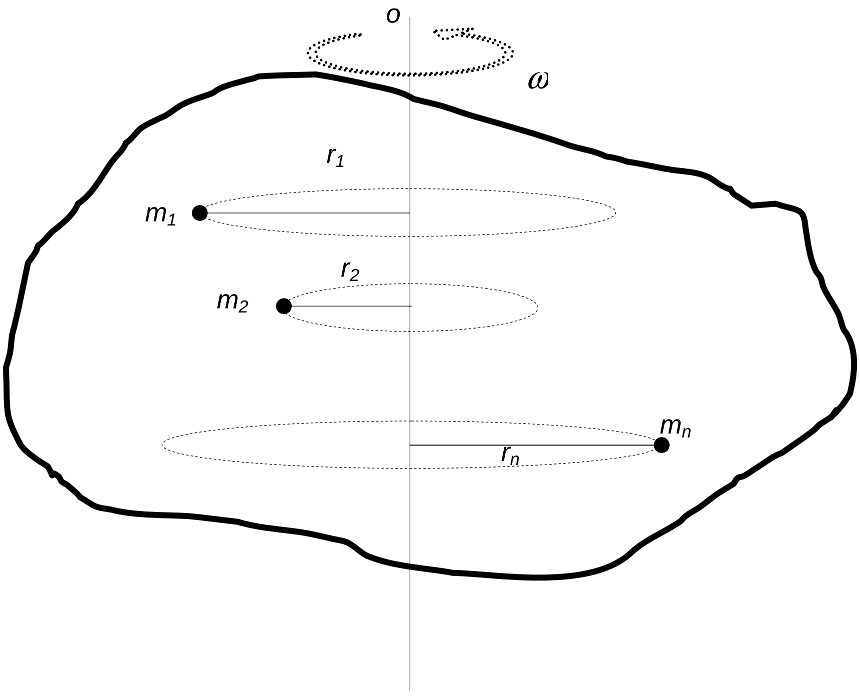
8. Kinetic energy of rotating objects

It needs some energy to start the rotation of a stationary object (force is needed to act over some distance – mechanical work is done) or to stop it when it is rotating. The energy depends on:

- ω
- mass of the object and also its distribution about the axis of rotation

Give some examples from everyday life to prove that.
Give the relation between ω and v: $v =$

One object can have kinetic energy due to its translation ($= \frac{1}{2}mv^2$) and kinetic energy due to rotation. We will derive the eqn.



$$E_{krot} = E_{k1} + E_{k2} + \ldots + E_{kn} = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + \ldots + \tfrac{1}{2}m_nv_n^2 = \tfrac{1}{2}\left(m_1\omega^2r_1^2 + m_2\omega^2r_2^2 + \ldots + m_n\omega^2r_n^2\right) =$$

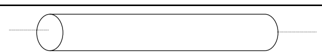
$$= \tfrac{1}{2}\omega^2\underbrace{\left(m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2\right)}_{J} = \tfrac{1}{2}J\omega^2$$

J ... moment of inertia

$$J = \sum_{i=1}^{n} m_i r_i^2$$

$$[J] = kg \cdot m^2$$

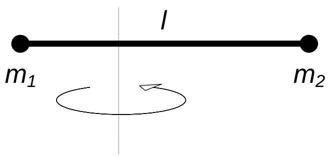
- depends on the mass of the object and the axis of rotation – more values for any object
- can be calculated for an object made of a reasonable number of parts (e.g. 3-4)
- for regular shapes – eqn in the book of data

shape	J	axis
full sphere	$\frac{2}{5}mr^2$	through the centre
empty sphere	$\frac{2}{3}mr^2$	through the centre
cylinder	$\frac{1}{2}mr^2$	

Use: Flywheels (e.g. in cars)

Questions:

11. Neglect the mass of the rod joining the small spheres (mass points) and calculate the moment of inertia and the kinetic energy due to rotation when the rod rotates at 10 rad·s^{-1} and the axis is perpendicular to the rod and a) passes the centre of mass of the system b) passes the centre of the rod.



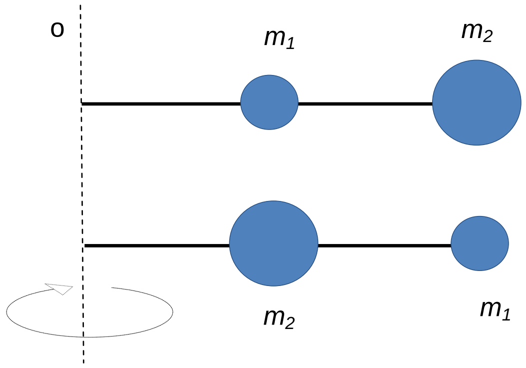
m_1 = 0.3 kg
l = 0.8 m
m_2 = 0.1 kg
ω =10 rad s^{-1}

12. Rotor of an electromotor has a moment of inertia of 1.2 kg·m^2 and it makes 50 revolutions per second. Calculate its kinetic energy.

13. Calculate kinetic energy of a full uniform cylinder of mass 5 kg and radius 0.2 m when it performs
a) translation at 6 m·s^{-1}
b) rotation so that the points on its surface move at 6 m·s^{-1}
c) rolls over the surface at 6 m·s^{-1}

14. Empty and full spheres have the same radius and mass. Which of them has bigger moment of inertia? Which of them is more difficult to rotate?

15. Assume two rods of negligible mass, with two balls of mass m_1 and m_2, m_1 = 210 g, m_2 = 420 g. The length of the rod is l = 140 cm and the inner ball is in the centre of the rod. State moments of inertia of both rods with respect to the axis o, that is perpendicular to them and goes through their endings. Which of them would have bigger kinetic energy if they rotated with the same angular velocity?



16. Stipulate the kinetic energy of the disc rotating around the axis and performing 400 revolutions per minute. The mass of the disc is m = 900 g, radius r = 50 cm and the moment of inertia with respect to the axis going through its centre is $\frac{1}{2}mr^2$.

L2/285-290, x291-3, 294-298

Answers:

1. 6.4 N; 0.28 Nm into
2. a) 0.4 Nm; b) 0; c) 0.4 Nm
3. d; c, b, e, d, a
5. 0.6 m
6. 8.7 cm
7. 64 cm from left
8. 2.43 m to the left from F_4
9. 45 J
10. turn
11. 0.048 kg·m^2, 2.4 J; 0.064 kg·m^2, 3.2 J
12. 59.2 kJ
13. a) 90 J; b) 45 J; c) 135 J
15. 0.926 kg·m^2, 0.617 kg·m^2
16. 100 J

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