

# ELECTRIC CHARGE AND ELECTRIC FIELD

## 1. Electric charge, electric force

An electric field is a space near charged objects or particles. It is a form of existence of matter just like any other kind of field.

An electric charge

- cannot exist without a particle or object, though we can speak about a „point charge“, it is in fact a very small charged particle
- can be moved from one object to another or even within one object
- conductors are materials in which the charge can move easily
- insulators or dielectrics are materials in which the charge cannot move at all or just very little
- there are two types of electric charge : positive and negative (traditional view)
- a glass rod rubbed by leather remains positively charged
- a plastic rod rubbed by fur remains negatively charged
- like charges repel, unlike charges attract
- the total charge of an isolated/insulated system remains the same = the law of conservation of charge
- movement of electrons  $\times$  movement of ions !  
matter itself doesn't move  $\times$  moves
- measuring instrument - electroscope or electrometer (with scale)
- a charge can be divided, the smallest amount is the charge of one electron = elementary charge
- $e = 1.602 \times 10^{-19}$  C
- the basic unit of charge is one coulomb (C) ,often used is a microcoulomb  $\mu\text{C} = 10^{-6}$  C

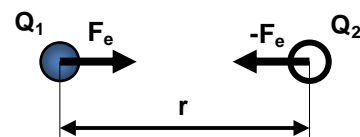
### Questions:

1. What happens if you move a negatively charged rod towards an electroscope?
2. What happens if you move a positively charged rod towards an electroscope?
3. What happens if you then move the rods away without the cap being touched?
4. What happens if you then move the rods away with the cap being touched?

## 2. Coulomb's law

- is about the force between two point charges  $Q_1$  and  $Q_2$ .

$$F_e = k \frac{Q_1 Q_2}{r^2} \quad (\text{size})$$



direction is attraction or repulsion according to the sign of the charges

the constant of proportionality  $k$  depends on the medium between the charges

$$k = \frac{1}{4\pi\epsilon_0\epsilon_r} = \frac{1}{4\pi\epsilon} \quad \text{for all media}$$

$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \cdot \text{m}^{-2} \cdot \text{N}^{-1}$  is the permittivity of vacuum/free space

$\epsilon_r$  is the relative permittivity of the given medium, values of  $\epsilon_r$  can be found in the book of data

medium	$\epsilon_r$
vacuum (and in practice air)	1
water	81

$k = 9 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2}$  ... for vacuum (and in practice air as well)

**Questions:**

5. Rewrite Coulomb's law with the complete equation for "k". Explain all the quantities, sketch the figure.
6. Two small spheres carry charges of -2 nC and 8 nC and they are 20 cm apart in the air. Calculate
  - a) the force between them
  - b) the force between the same charges when the distance is doubled
  - c) the force between the charges when we pour oil of relative permittivity 6 between the charges
  - d) the force between the spheres when we put them in contact and then move apart to the distance of 20 cm.
7. How many elementary charges represent 1 μC?
8. If the force between two charged bodies is  $F$ , then doubling the charges carried by both bodies and doubling their distance apart changes the force to
  - a)  $\frac{F}{4}$
  - b)  $\frac{F}{2}$
  - c)  $F$
  - d)  $2F$
  - e)  $4F$

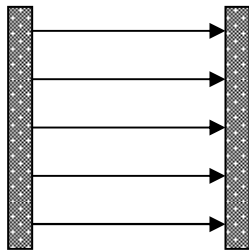
**3. Models of electric field**

electric field lines =

according to the direction of force that would be acting on a ..... charge  
(state the polarity of the desks and point charges)

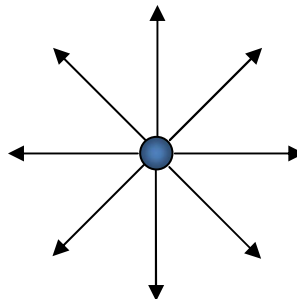
**types of electric field**

uniform

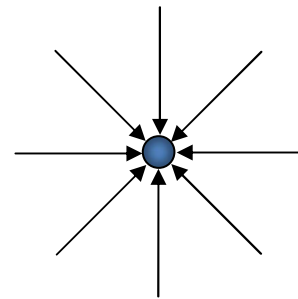


central

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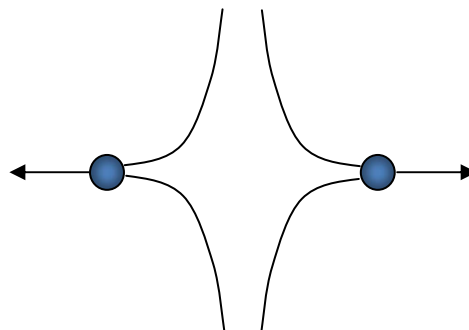


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combined electric fields of two charges

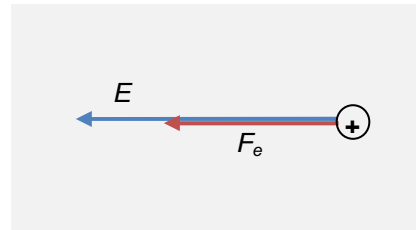
(finish the figures of the fields around the charges, state the polarity of the charges)



#### 4. Electric field strength $\vec{E}$

is a vector quantity used to determine the electric field

$$\vec{E} = \frac{\vec{F}_e}{q}$$



$\vec{F}_e$  ... electric force acting at a certain point of the el. field on a test charge  $q$   
 $q$  ... test charge

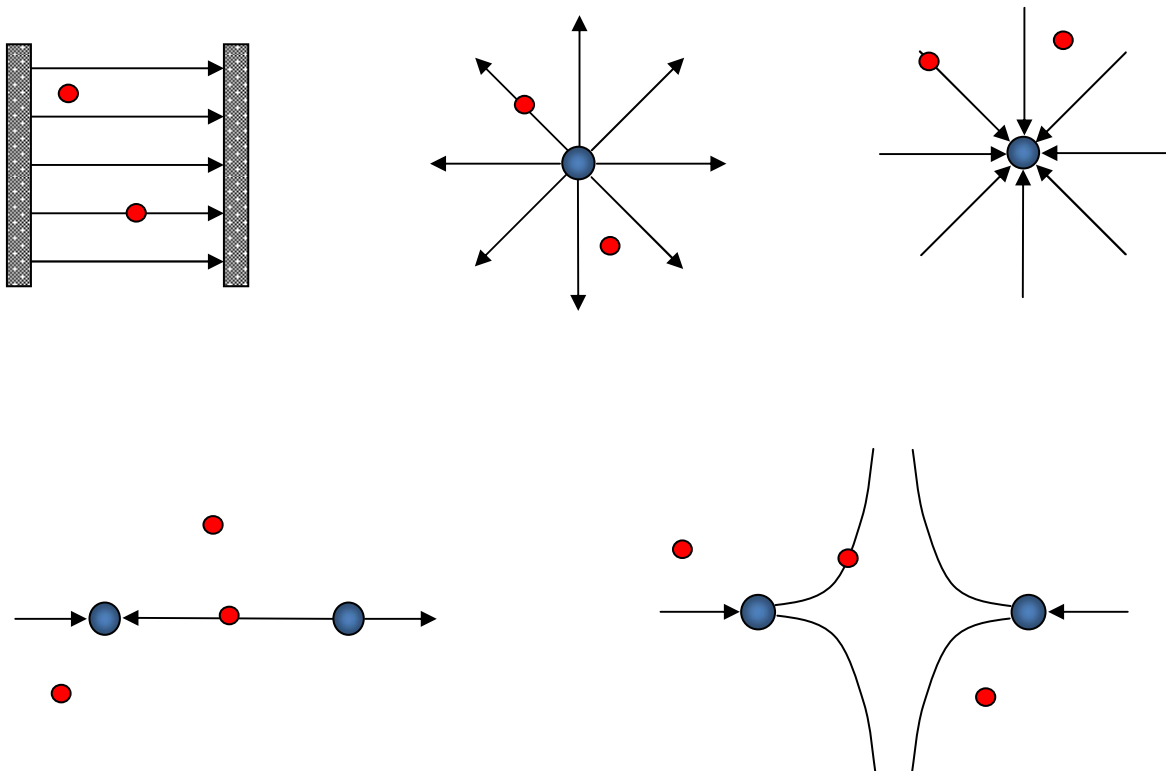
$$[\vec{E}] = \text{N} \cdot \text{C}^{-1} \text{ (or } \text{V} \cdot \text{m}^{-1} \text{)}$$

for the size of  $\vec{E}$ :  $E = \frac{F_e}{q} = k \frac{|Qq|}{r^2} \cdot \frac{1}{q} = k \frac{|Q|}{r^2}$

The direction of the electric field strength at a point varies according to the direction of the electric force which would be acting on a positive test charge at this point. It is also at a tangent to the electric field lines which represent a theoretical trajectory of a positive test charge in this field.

#### Questions:

9. Sketch the electric field strength vectors from the points in the following figures



## 5. Electric potential $\varphi$

is a scalar quantity used to describe the electric field

$$\varphi_A = \frac{W_{0A}}{q}$$

$\varphi_A$  ... electric potential at a point A in the electric field

$W_{0A}$  ... work done by electric forces to move  $q$  from A to zero level of el. potential (Earth, infinity)

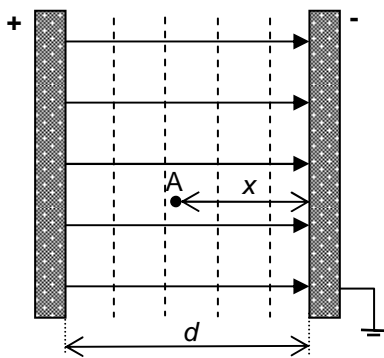
$q$  ... test charge

$$[\varphi] = \text{J} \cdot \text{C}^{-1} = \text{V}$$

Electric potential at some point in the el. field equals 1V when electric work of 1J is done to move the charge of +1C from this point to zero level of the el. potential

equipotentials = surfaces with the same value of el. potential

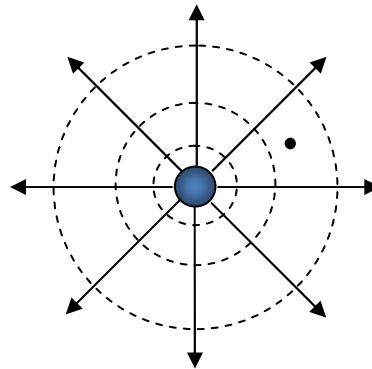
uniform el. field



$$\varphi_A = \frac{W_{0A}}{q}$$

$$\varphi_A = Ex$$

central el. field



$$\varphi_A = \frac{W_{0A}}{q}$$

$$\varphi_A = k \frac{Q}{r}$$

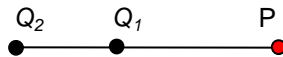
### Questions:

10. Two point charges  $Q_1 = 5 \text{ nC}$  and  $Q_2 = -8 \text{ nC}$  are placed in a vacuum. Calculate the potential and state the field strength at a point P that is 0.5 m from the first charge and 120 cm from the second one as shown on the figures.

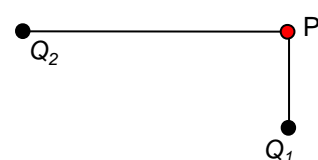
a)



b)



c)



11. Calculate potential and field strength in similar situations a), b), and c) just for both charges having positive values and placed in a medium of relative permittivity 30.

L5/1-30

## 6. Work done in electric field

in any type of field - equation

$$W = \int F ds \quad \text{varies according to the type of field}$$

( $F \neq \text{const}$  in the central el. field !!!)

uniform electric field

$$W = F_e d \cos \alpha = qEd \cos \alpha$$

$W$ ... work done to move  $q$  from A to B

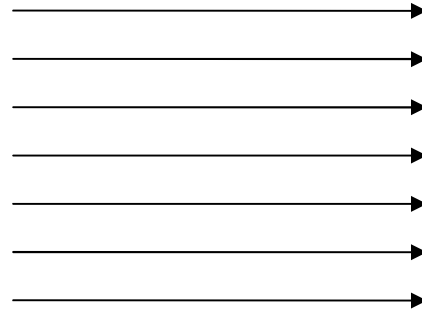
$E$  ... value of electric field strength

$d$  ... distance between A and B

$\alpha$  ... angle between the force on  $q$  and the displacement

$W > 0$  when done by the forces of the field

$W < 0$  when done by outer force against the field



voltage (potential difference)

between two points A and B is the work done to move the charge of +1C from A to B = difference in the potentials of the points = potential difference

$$U_{AB} = \frac{W_{AB}}{q} = \varphi_B - \varphi_A \quad \text{general eqn for any field}$$

See figure for equipotentials of the uniform el. field :

What is the relation between the voltage between the plates and the electric field strength?

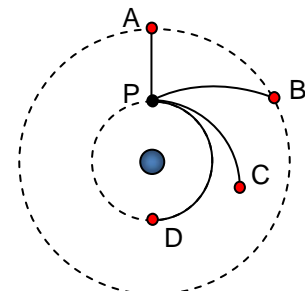
$$U = \varphi_+ - \varphi_- = \varphi_+ - 0 = W_{+0} = \frac{F_e d}{q} = \frac{Eqd}{q} = Ed$$

$$[E] = \left[ \frac{U}{d} \right] = \text{V} \cdot \text{m}^{-1} \quad \text{another unit of } \dots\dots\dots$$

### Questions:

12. A charge of  $20 \mu\text{C}$  is moved in the uniform electric field of strength  $20 \text{ V} \cdot \text{m}^{-1}$  by 50 cm. Calculate
  - a) voltage between the plates when the distance between them is 120 cm
  - b) work done when the charge is moved in the direction of the field lines
  - c) work done when the charge is moved against the direction of the field lines
  - d) work done when the charge is moved perpendicularly to the field lines
  - e) which of the answers a)-d) will change when the charge is negative and what will be the new results

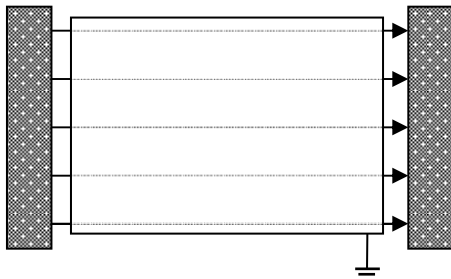
13. The point source creates a central electric field as shown. To which of the points A, B, C and D should be moved the test charge along the given trajectories from P to do
  - a) maximum work
  - b) minimum work



14. The first body has a potential of 100 V and the second one -20 V. What is the value of the charge moved from the first one to the second one when the work done is 6 mJ?
15. A uniform electric field between two parallel plates 20 cm apart has the value of  $400 \text{ N}\cdot\text{C}^{-1}$ . Calculate:
- voltage between the plates
  - the force that would be acting on a charge of -2 nC placed in the field
  - what would be the electric field strength when the distance between the plates changes to 30 cm?

## 7. Conductor and insulator in electric field, distribution of charge

### Conductor



When we place a conductor into the electric field free electrons will move according to the field and different parts of the object become oppositely charged though the total charge of the object remains zero. The effect is called electrostatic induction. When we earth the object as on the figure it can suck free electrons from the Earth and if we then disconnect it, we can obtain a negatively charged object. When we earth the opposite part of the object, we of course get a positively charged object.

### Distribution of charge on the conducting object

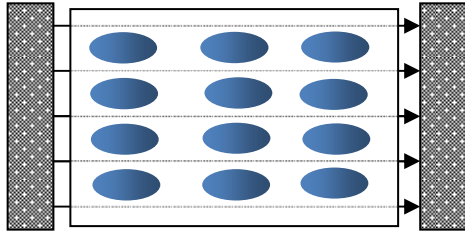
$$\sigma = \frac{Q}{A} \text{ charge density}$$

$$[\sigma] = \text{C} \cdot \text{m}^{-2}$$

explain with the help of experiments seen that the charge is distributed over the outer surface only

with maximum  $\sigma$  on convex regions – peaks, tips

### Insulator



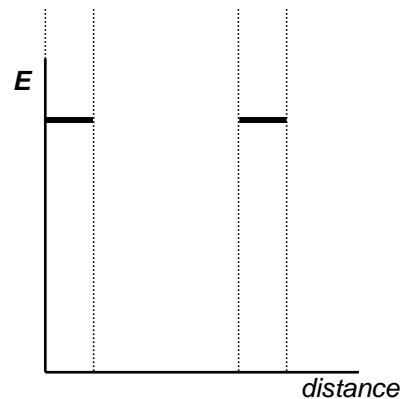
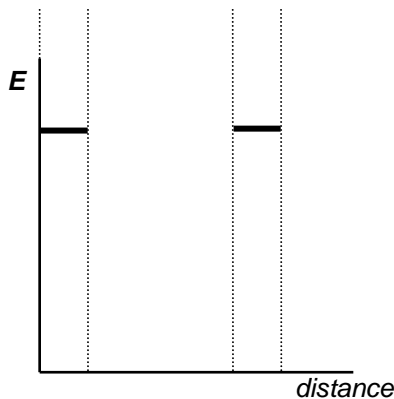
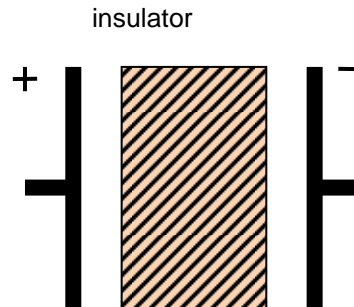
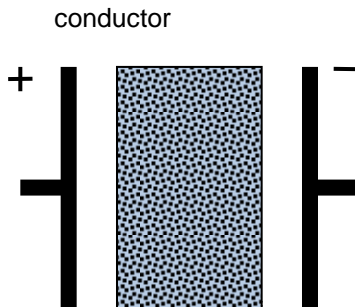
Insulator or dielectrics is a substance without free charged particles. When we place it into the outer electric field its molecules can turn (e.g. in polar liquids like water) or the electron shells can be deflected so that an opposite charge is obtained as on the figure (e.g. bits of paper). The inner electric field is therefore pointed against the outer one and it varies according to the type of dielectrics. The effect is called polarisation of dielectrics



$$\epsilon_r = \frac{E}{E - E_i} = \frac{\text{outer e.f.s.}}{\text{resultant e.f.s.}} = \text{relative permittivity of the medium}$$

### Questions:

16. Can we obtain two pieces of oppositely charged material when we place in the outer electric field
  - a) a piece of metal
  - b) a piece of polystyrene? Explain.
17. What will you observe when a piece of metal is placed into the electric field? Is there any attraction or repulsion?
18. What if you do the same with a piece of insulator? Is it the same or not? What is the difference?
19. Estimate the value of resultant electric field strength inside a conductor or insulator of relative permittivity 4 and finish the graphs



## 8. Uses of electrostatics

### Smoke cleaning

<http://www.explainthatstuff.com/electrostaticsmokeprecipitators.html>

principle:

### Copier

<http://home.howstuffworks.com/photocopier.htm>

<http://en.wikipedia.org/wiki/Photocopier>

<http://projektysipvz.gytool.cz/ProjektySIPVZ/Default.aspx?uid=244>

parts:

principle:

analogue x digital:



## 9. Capacitance (C)

is the ability of a conductor to store a charge.

$$C = \frac{Q}{\Delta\varphi}$$

C ... capacitance of the conductor

Q ... charge supplied to the conductor

$\Delta\varphi$  ... change of its potential

$$[C] = C \cdot V^{-1} = F \quad (\text{farad})$$

$$C = 4\pi\epsilon_0 R \quad \text{capacitance of a conducting sphere}$$

capacitance of isolated objects is very small, e.g. for  $R = 10 \text{ cm}$ ,  $C = 11 \text{ pF} = 11 \times 10^{-12} \text{ F}$

### Question:

20. State the radius of a sphere having the capacitance of 1 F.

## 10. Capacitors

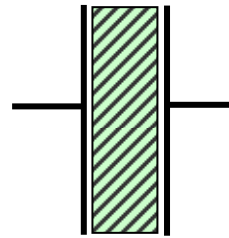
are special components used to store charge. They consist of two parallel plates separated by an insulator.

$$C = \epsilon_0 \epsilon_r \frac{A}{d}$$

A ... common area of the plates

d ... distance .....

$\epsilon_r$  ... relative permittivity of .....



$$C = \frac{Q}{U}$$

[http://www.ngsir.netfirms.com/englishhtm/RC\\_dc.htm](http://www.ngsir.netfirms.com/englishhtm/RC_dc.htm)

**Types of capacitors** - according to the medium between the plates

air variable -  $\Delta A \Rightarrow \Delta C$

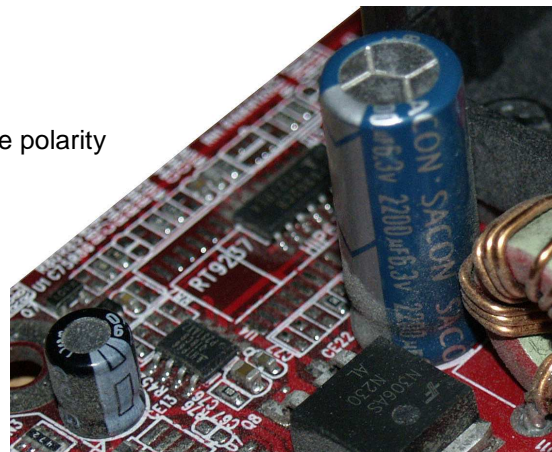
mica

ceramic – big capacitance related to size

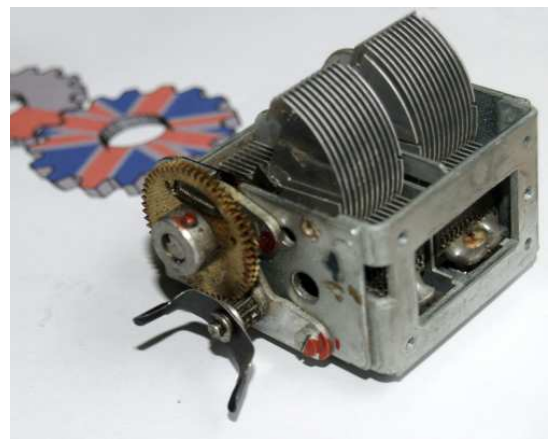
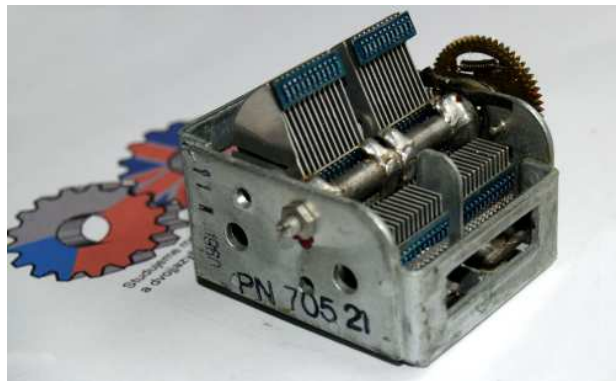
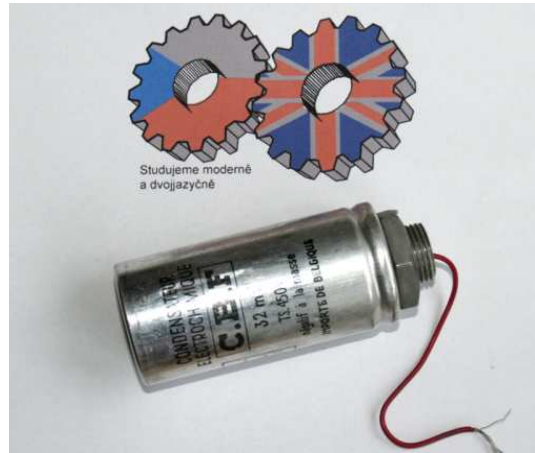
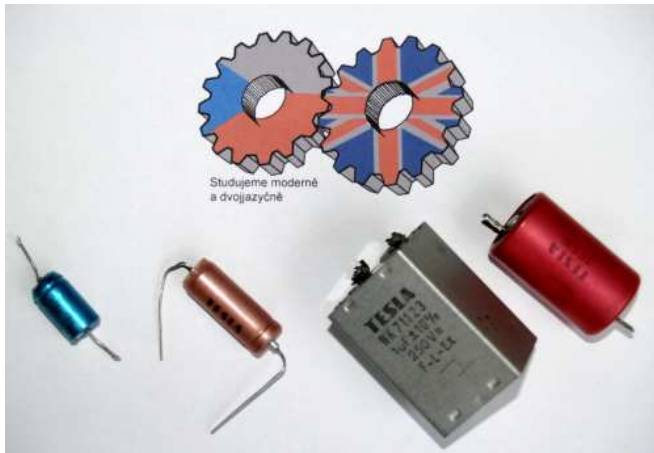
electrolytic – can be used in d.c.circuits in one polarity

waxed paper

plastic



What types of capacitors are on the figures below?



## Capacitor networks

Capacitors in parallel always have a bigger total capacitance than the component capacitors; we are in fact making a capacitor with a bigger plate area.

$$C = C_1 + C_2 + \dots + C_n$$

Capacitors in series have always smaller total capacitance than any of the component capacitors. The total charge stored is the same as the charge on any of the capacitors. The voltage is divided as if there are resistors.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

### Questions:

21. Three capacitors of capacitance 1 nF, 2 nF and 4 nF are connected to a source of 10 V. Calculate the charge and voltage on each when they are connected:

- in parallel
- in series
- 1 nF capacitor connected in series to the parallel connection of the other two
- 4 nF capacitor connected in parallel to the series connection of the other two

22. Three capacitors of capacitances 1 nF, 2 nF and 4 nF are connected

- in series
- in parallel

Discuss where will be the biggest amount of the total charge and voltage in each of the connections a) and b)

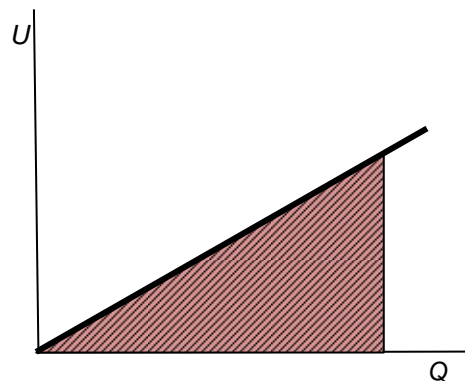
## Energy of a charged capacitor

During the charging of a capacitor the charge moves to the plates due to the work done by electric forces. That is why the capacitor gains the same value of energy that can be used during the discharge e.g. to light the bulb of a "forever flash". The energy is stored in the polarized dielectrics, in the electric field between the plates. The equation shows that assuming constant value of capacitance, the charge stored is directly proportional to the voltage between the plates:

$C = \frac{Q}{U}$  As we define the voltage  $U = \frac{W}{Q}$ , we can get the equation for work and hence energy

$W = \int U dQ$  This could be easily calculated as the area under the U – Q graph below.

$$W = \int U dQ = \frac{1}{2} UQ = \frac{1}{2} U^2 C = \frac{1}{2} \frac{Q^2}{C}$$



**Questions:**

23. A  $10 \mu\text{F}$  capacitor is connected to a  $50 \text{ V}$  supply. Calculate the charge and energy stored on the capacitor.

L5/41-57

**Answers:**

6. a)  $3.6 \mu\text{N}$  attraction    b)  $0.9 \mu\text{N}$     c)  $0.6 \mu\text{N}$     d)  $2.025 \mu\text{N}$  repulsion  
7.  $6.24 \times 10^{12} e$   
8. c)  
10. a)  $30 \text{ V}; 230 \text{ N}\cdot\text{C}^{-1}$     b)  $30 \text{ V}; 130 \text{ N}\cdot\text{C}^{-1}$     c)  $30 \text{ V}; 187 \text{ N}\cdot\text{C}^{-1}$   
11. a)  $5 \text{ V}; 4.3 \text{ V}\cdot\text{m}^{-1}$     b)  $5 \text{ V}; 7.6 \text{ V}\cdot\text{m}^{-1}$     c)  $5 \text{ V}; 6.2 \text{ V}\cdot\text{m}^{-1}$   
12. a)  $24 \text{ V}$     b)  $0.2 \text{ mJ}$     c)  $-0.2 \text{ mJ}$     d)  $0$     e) b)  $-0.2 \text{ mJ}$ ; c)  $0.2 \text{ mJ}$   
13. a) A and B    b) D  
14.  $50 \mu\text{C}$   
15. a)  $80 \text{ V}$     b)  $0.8 \mu\text{C}$     c)  $267 \text{ N}\cdot\text{C}^{-1}$   
20.  $9 \times 10^9 \text{ m}$   
21.    a)  $10 \text{ nC}, 20 \text{ nC}, 40 \text{ nC}, 10 \text{ V}, 10 \text{ V}, 10 \text{ V}$   
      b)  $5.7 \text{ nC}, 5.7 \text{ nC}, 5.7 \text{ nC}, 5.7 \text{ V}, 2.9 \text{ V}, 1.4 \text{ V}$   
      c)  $8.57 \text{ nC}, 2.86 \text{ nC}, 5.71 \text{ nC}, 8.57 \text{ V}, 1.43 \text{ V}, 1.43 \text{ V}$   
      d)  $6.7 \text{ nC}, 6.7 \text{ nC}, 40 \text{ nC}, 6.7 \text{ V}, 3.3 \text{ V}, 10 \text{ V}$   
23.  $0.5 \text{ mC}, 12.5 \text{ mJ}$