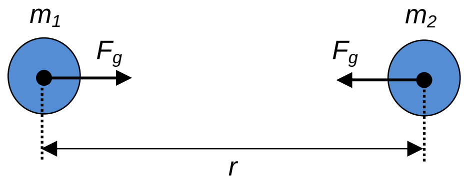
EUROPEAN UNION
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitivness
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

GRAVITATIONAL FIELD

1. The Law of Universal Gravitation

(1687 Newton in The Principia)

Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of their masses and indirectly proportional to the square of their distance apart.



$$F_g \propto \frac{m_1 m_2}{r^2}$$

$$F_g = G\frac{m_1 m_2}{r^2}$$

G (κ) ... universal gravitational constant (stated by Cavendish later)

size of the mutual gravitational attraction

$G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2\cdot\text{kg}^{-2}$

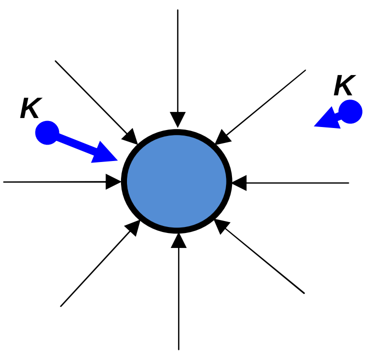
Questions:

1. What is the gravitational attraction between the Earth and the Moon?

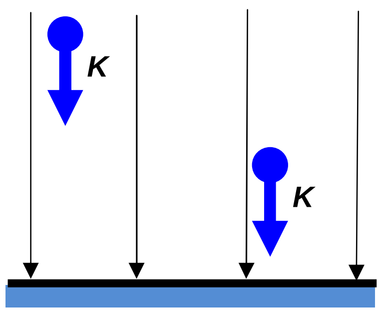
2. The gravitational attraction between two objects is 2×10^{-5} N. What is the new force between the objects
   a) when their distance is doubled
   b) when the mass of each object is doubled

- models of gravitational field

central



uniform



Which model is suitable for which problems? Discuss.

Centrální (radiální) pole je gravitační. **Gravitační síla F_g** působící na těleso v různých místech tohoto pole je různá. Homogenní pole v okolí Země nazýváme tíhové. **Tíhová síla F_G** přitahující těleso k Zemi je ve všech místech tohoto pole stejná (proč?).

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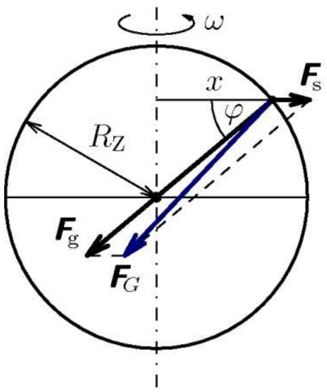
Ve všech místech gravitačního pole Země směřuje gravitační síla $\boldsymbol{F_g}$ ($\boldsymbol{a_g}$ i $\boldsymbol{K}$) do středu Země. Takové pole se nazývá **centrální gravitační pole.** Gravitační síla $\boldsymbol{F_g}$ působící na těleso v různých místech tohoto pole je různá. Centrální gravitační pole je v okolí každého stejnorodého tělesa, které má tvar koule (i v okolí hmotného bodu). Centrální gravitační pole je prostorově neohraničené.

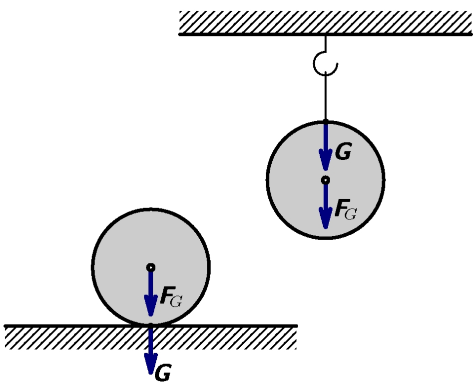
Sledujeme-li účinky gravitační síly v malých oblastech gravitačního pole Země (např. při povrchu Země v rozmezí několik set metrů), zjistíme, že se v jednotlivých bodech gravitační síly $\boldsymbol{F_g}$ ($\boldsymbol{a_g}$ i $\boldsymbol{K}$) odlišují jen nepatrně, a to co do velikosti, tak co do směru. Takové pole pak nazýváme **homogenní gravitační pole.**

Pohyby těles obvykle vztahujeme k zemskému povrchu, který považujeme za inerciální vztažnou soustavu. Ve skutečnosti však povrch Země tvoří s ohledem na rotaci Země kolem své osy

neinerciální vztažnou soustavu (se stálou úhlovou rychlostí $\omega = 2\pi / T$, kde T je doba jednoho otočení Země). V této neinerciální vztažné soustavě působí na všechna tělesa při povrchu Země, která neleží na ose otáčení, kromě gravitační síly $\boldsymbol{F_g}$, směřující do gravitačního středu, ještě setrvačná odstředivá síla $\boldsymbol{F_s}$, směřující kolmo od osy otáčení. Výslednice obou sil je **tíhová síla** $\boldsymbol{F_G}$.
Tíhová síla je vektorovým součtem gravitační síly $\boldsymbol{F_g}$ a setrvačné odstředivé síly $\boldsymbol{F_s}$.

Tedy $\vec{F}_G = \vec{F}_g + \vec{F}_s$. Prostor při povrchu Země, v němž se projevují účinky tíhové síly, nazýváme **tíhové pole**.





2. Gravitational field strength ($\vec{K}$, K)

is a vector quantity used to describe the gravitational field.

definition eqn $\vec{K} = \dfrac{\vec{F}_g}{m}$

unit $[K] = \mathrm{N \cdot kg^{-1}}$

for the SIZE of $\vec{K}$: $K = \dfrac{F_g}{m} = G\dfrac{M\,m}{r^2\,m} = G\dfrac{M}{r^2}$

direction of $\vec{K}$ - tangent to the gravitational field lines - see the models above

3. Acceleration due to gravity ($\vec{a}_g$, a_g)

We can compare the definition eqn. for the gravitational field strength and the Newton's 2nd Law:

$$\vec{K} = \frac{\vec{F}_g}{m} \qquad\qquad \vec{F} = m\,\vec{a}$$

$$\vec{F}_g = m\vec{K} \qquad\qquad \vec{F}_g = m\,\vec{a}_g$$

From that $\vec{K} = \vec{a}_g$, so the acceleration due to gravity given to the test mass *m* at certain point in the gravitational field equals the gravitational field strength at this point.

Let's check the units. The unit of *K* is $\mathrm{N \cdot kg^{-1}}$, but the unit of any acceleration is $\mathrm{m \cdot s^{-2}}$.

$$\mathrm{N \cdot kg^{-1}} = \frac{\mathrm{N}}{\mathrm{kg}} = \frac{\mathrm{kg \cdot m \cdot s^{-2}}}{\mathrm{kg}} = \mathrm{m \cdot s^{-2}}$$ so the units are equivalent

How much is the acceleration due to gravity on the Earth's surface and how does it vary with the distance from its surface?

(size) $a_{g0} = G\frac{M_E}{R_E^2} \cong 9.83\,\mathrm{m \cdot s^{-2}}$ on the surface

(size) $a_{gh} = G\frac{M_E}{(R_E + h)^2}$ for the point *h* metres from the surface

Gravitační zrychlení $\boldsymbol{a_g}$ používáme v centrálním gravitačním poli, jeho hodnota je závislá na místě, ve kterém ho určujeme. **Tíhové zrychlení *g*** je vektorový součet gravitačního a odstředivého zrychlení a souvisí s tíhovou silou $\boldsymbol{F_G = mg}$. Má různou hodnotu pro různá místa na Zemi (viz dynamika), dohodou bylo zavedeno normální tíhové zrychlení $\boldsymbol{g}_n = 9.80665\ \mathrm{m \cdot s^{-2}}$, kterou často zaokrouhlujeme na $10\ \mathrm{m \cdot s^{-2}}$.

4. Gravitational potential φ

is another quantity used to describe gravitational fields, it is a scalar

$$\varphi = \frac{W}{m} = \frac{E_p}{m}$$

- φ ... gravitational potential at a certain point in the gravitational field
- *m* ... mass of a test mass
- *W*... work done to move the test mass from infinity or zero level of potential to the point
- E_p ... potential energy at the point

$$[\varphi] = \mathrm{J \cdot kg^{-1}}$$

<u>equipotentials</u> are surfaces at which the gravitational potential is the same (spheres, planes, ..)
they are always perpendicular to the gravitational field lines

sketch the models with field lines, equipotentials and field strength

central field **uniform field**

Sun: mass 2×10^{30} kg; mean distance of the centres: Sun – Earth 150×10^{6} km
Earth: mass 6×10^{24} kg, radius 6378 km
Moon: mass 7.4×10^{22} kg, radius 1 700 km; mean distance of the centres: Moon – Earth 3.8×10^{8} m

Questions:

3. An astronaut of mass 75 kg on the Moon is attracted towards its centre by a force of 126 N. Calculate the value of the gravitational field strength on the Moon's surface.

4. The value of *K* on the Earth's surface is about 10 N·kg^{-1}. State the value of the gravitational field strength at the height *h* above the surface when a) $h = R_E$ b) $h = 3R_E$

5. State the acceleration due to gravity which acts on the Moon because of the attraction by the Earth.

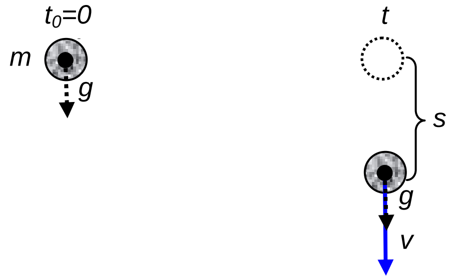
6. Calculate the value of acceleration due to gravity on the Moon's surface.

L2/217,219,220-1, 223-4

5. Motion in the uniform gravitational field

a) <u>free fall</u> (revision)

- in a ....
- in the gravitational field NEAR the Earth's surface
  $\boldsymbol{a} = \boldsymbol{g}$ = 9.81 m·s^{-2} = 10 m·s^{-2}
- from .... (v_0=0)



distance fallen: $s = s_0 + v_0 t + \frac{1}{2}at^2 = \frac{1}{2}gt^2$

speed of falling object : $v = v_0 \pm at = gt$

b) <u>projectile motion</u>

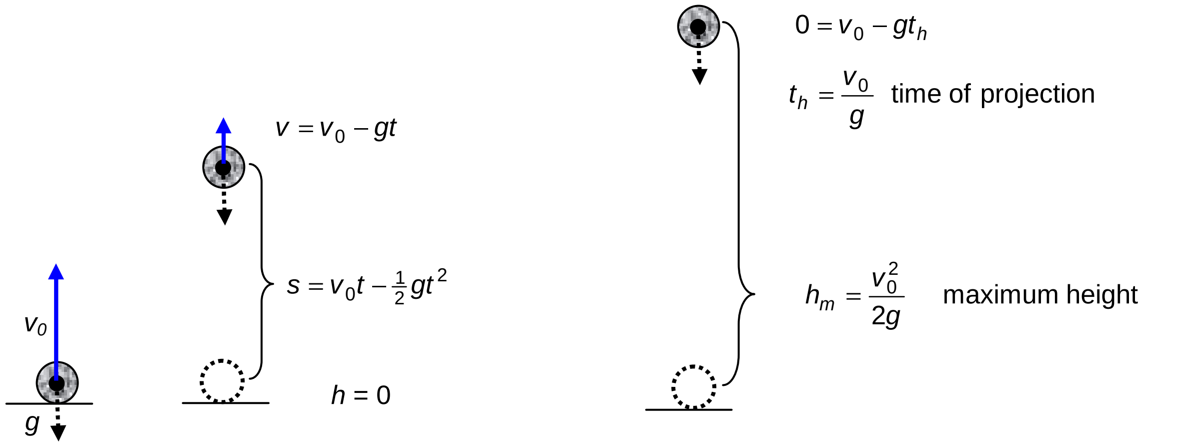
= free fall (direction ↓) + steady linear motion (direction $\vec{v}_0$)

- in a vacuum
- ***a = g***
- with initial speed $\boldsymbol{v_0} \neq 0$

i) vertical projection

we will discuss vertical upward projection (steadily decelerated motion), because the downward one is just steadily accelerated motion with initial speed

$t_0 = 0$ $\quad$ t (any time when moving up) $\quad$ $t = t_h$ (the object stops at the max. height)



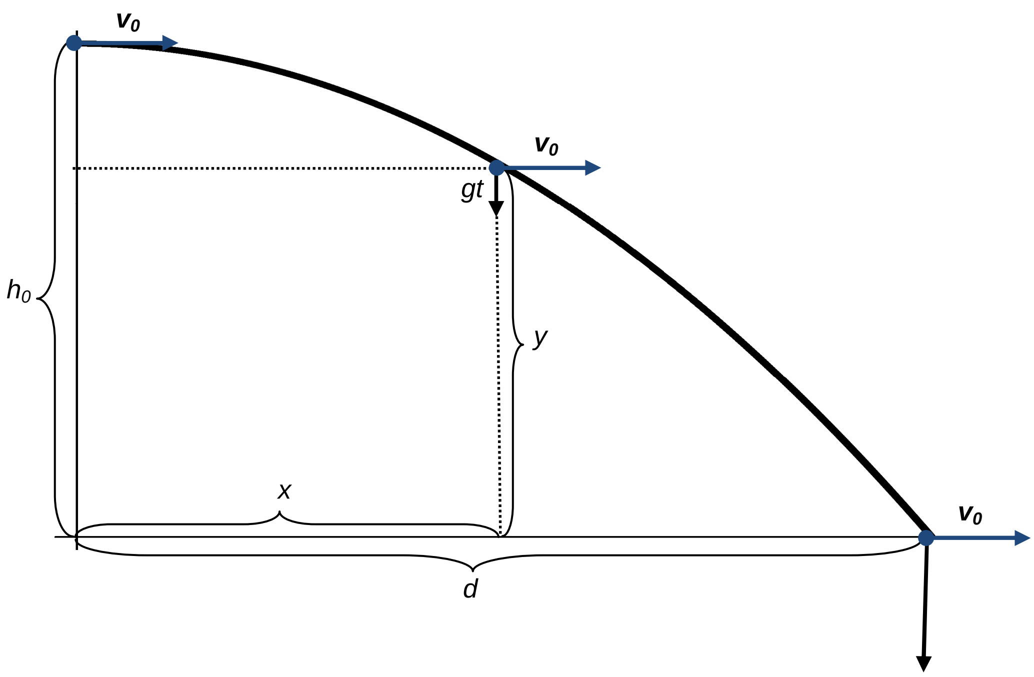
Questions:

7. A bullet is fired vertically with a speed of 40 m·s^{-1}. Calculate the speed and distance travelled 2 seconds after the beginning of the motion, the maximum height reached and the time taken to get there. How long will it take to hit the ground again? Sketch the speed-time and velocity-time graphs. Find the differences, discuss.

L2/231-6, X237

ii) horizontal projection
can be divided into two types of motion in perpendicular direction – free fall from the „side“ and steady motion as the „upper view“

$t_0 = 0$ t t_d



t (any time of movement)

$$y = h_0 - s = h_0 - \tfrac{1}{2} g t^2 \qquad x = v_0 t \qquad v = \sqrt{v_0^2 + (gt)^2} \quad \tan\alpha = \frac{gt}{v_0}$$

t_d (time of projection)

$$h_0 = \frac{1}{2} g t_d^2 \quad \Rightarrow \quad t_d = \sqrt{\frac{2h_0}{g}}$$

$$d = v_0 t_d = v_0 \sqrt{\frac{2h_0}{g}} \qquad v = \sqrt{v_0^2 + (g t_d)^2} \qquad \tan\alpha = \frac{g t_d}{v_0}$$

Questions:
8. A car manufacturer tests crash resistance by driving test vehicles off a horizontal platform so that they fall to a concrete surface below. If the car is driven off at 20 m·s^{-1} and the platform is 15 m above the ground calculate the impact angle and speed, and the horizontal distance of the impact from the edge of the platform.

9. An object is dropped from a helicopter at a height of 55 m above the ground. a) If the helicopter is at rest, how long does the object take to reach the ground and what is its velocity on impact? b) If the helicopter is falling with a velocity of 1m·s^{-1} when the object is released, what will be the final velocity of the object? Assume g = 9.81 m·s^{-2}. The result must be rounded to 2 decimal points.

10. An object is released from an aircraft travelling horizontally with a constant velocity of 150 m·s^{-1} at a height of 800 m. Ignoring air resistance and taking g = 10 m·s^{-2} find a) how long it takes the object to reach the ground, b) the horizontal distance covered by the object between leaving the aircraft and reaching the ground.

11. An arrow of mass 10 g is shot horizontally at 25 m·s^{-1} from a tower at a height 80 m. a) How long does the arrow take to reach the horizontal surface of the surrounding terrain and how far from the bottom of the tower does it fall? b) What will be the kinetic and potential energy of the arrow at the beginning of the motion? c) What is the total mechanical energy of the arrow during the motion?

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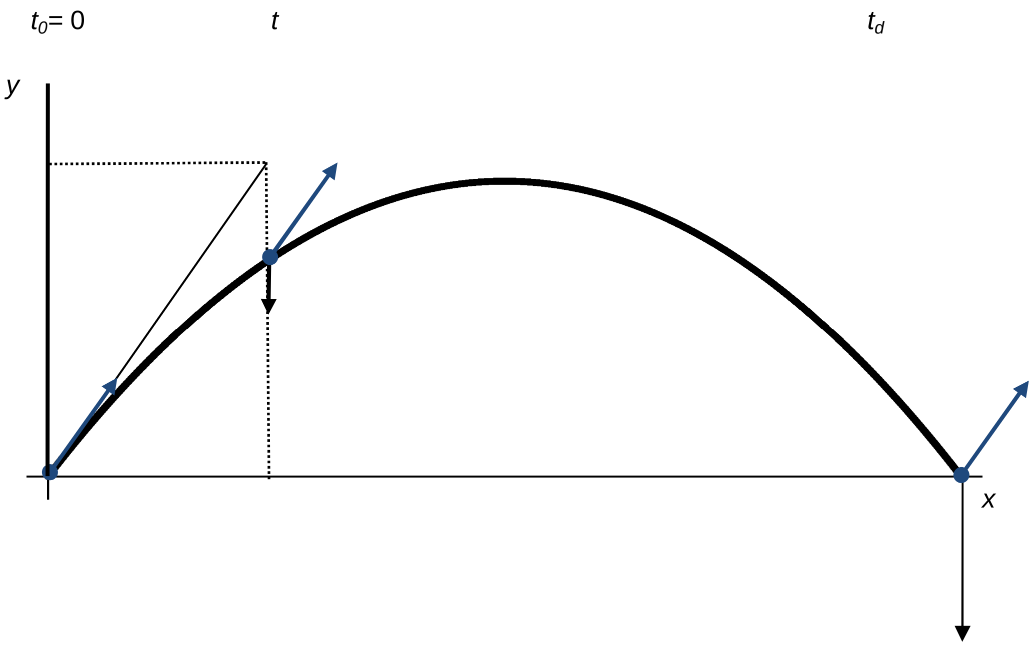
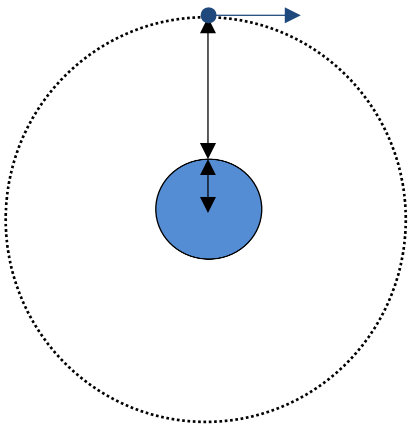
iii) projection at angle



Questions:
12. A footballer kicks a ball from the pitch at an angle of 45°. The ball fell down at the distance 40 m from the point of kicking off. What was its initial speed?

6. Motion in the central gravitational field - artificial satellites

- circular trajectory – steady circular motion
- the centripetal force = the force of gravity



$$F_c = F_g$$

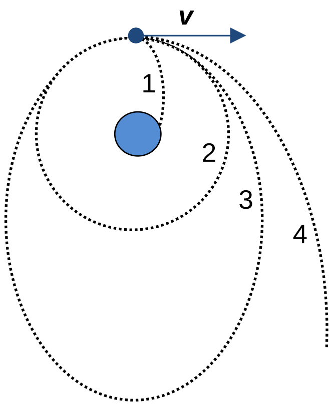
$$m\frac{v_c^2}{r} = G\frac{mM_E}{r^2}$$

$$v_c = \sqrt{G\frac{M_E}{r}} = \sqrt{G\frac{M_E}{R_E + h}}$$

$$h \uparrow \Rightarrow v_c \downarrow$$

Questions:
13. Calculate the circular speed for $h \langle\langle R_E$ (e.g. 20 km)
(1. kosmická rychlost, 2. kosmická rychlost je pro opuštění grav. pole Země – těleso se stane oběžnicí kolem Slunce, je to $\sqrt{2}v_c$).

- trajectory and speed



1. $v \langle v_c$ (part of) an ellipse
2. $v = v_c$ a circle
3. $v_c \langle v \langle \sqrt{2}v_c$ an ellipse
4. $v = \sqrt{2}v_c$ a hyperbola
5. $v \rangle \sqrt{2}v_c$ a parabola - satellite of the Sun

Questions:
14. Calculate the height of a geostationary satellite.

L2/246-252, 254, 256-7

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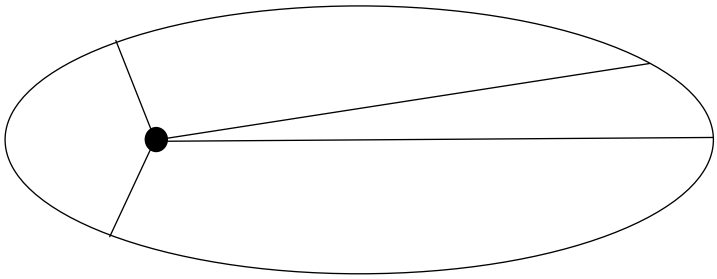
7. Planetary motion in the Solar system

Kepler's laws (1473-1543)

1. Each planet moves in an ellipse that has the Sun at one focus.
   (The trajectories are almost circles)

2. The line joining the Sun to the planet sweeps out equal areas in equal times.
   ($\Rightarrow$ closer - faster)



3. The squares of times of revolution of the moving planet about the Sun are proportional to the cubes of its major axis.

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

period of revolution and major axis of e.g. Mercury — the same for e.g. Venus (Earth, Jupiter, ...)

This is valid for ANY two objects revolving around the central one.

Prove the 3rd K.L. using Newton's Law of Universal Gravitation

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Fill in the following table using different resources:

planet	mean distance m	period of revolution s	$\frac{T^2}{r^3}$
Mercury			
Venus			
Earth			
Mars			
Jupiter			
Saturn			
Uranus			
Neptune			

8. The Solar System

about ....milliard years old

The Sun

represents .... % of the mass of the Solar System, about .... mass of the Earth

surface temperature

inner temperature

structure

Inner planets

Mercury

Venus

Earth

Mars

Band of asteroids

Outer planets

Jupiter

Saturn

Uranus

Neptune

Comets

nucleus

coma

tail

Answers:

1. 2×10^{20} N
2. a) 5×10^{-6} N b) 8×10^{-5} N
3. 1.68 $N\cdot kg^{-1}$
4. a) 2.5 $N\cdot kg^{-1}$ b) 0.625 $N\cdot kg^{-1}$
5. 2.7×10^{-3} $m\cdot s^{-2}$
6. 1.7 $N\cdot kg^{-1}$
7. a) 20 $m\cdot s^{-1}$, 60 m b) 80 m, 4 s, 8 s
8. 41°, 26.46 $m\cdot s^{-1}$, 34.64 m
9. a) 3.35 s; 32.85 $m\cdot s^{-1}$ b) 3.25 s; 32.86 $m\cdot s^{-1}$
10. a) 12.65 s b) 1897 m
11. a) 4 s, 100 m b) 3.1 J; 7.9 J c) 11 J
12. 20 $m\cdot s^{-1}$
13. 7.9 $km\cdot s^{-1}$
14. 36 000 km

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