

# STRUCTURE AND PROPERTIES OF GASES

## 1. Ideal gas

represents an ideal model of a gas which is often used to describe some situations. Later on we e.g. assume that all the molecules have ONE speed only, which is „typical“ for the sample (= r.m.s. speed, neither average nor the most probable – see later)

IDEAL GAS:

- Sizes of molecules are negligible related to the mean distance between them ( $\Rightarrow$  not many molecules, compressible to 0 volume)
- The only forces between the molecules are because of their collisions
- The collisions (molec.  $\times$  molec., molec.  $\times$  walls of the container) are ideally elastic ( $\Rightarrow$  no kinetic energy converted into other types)

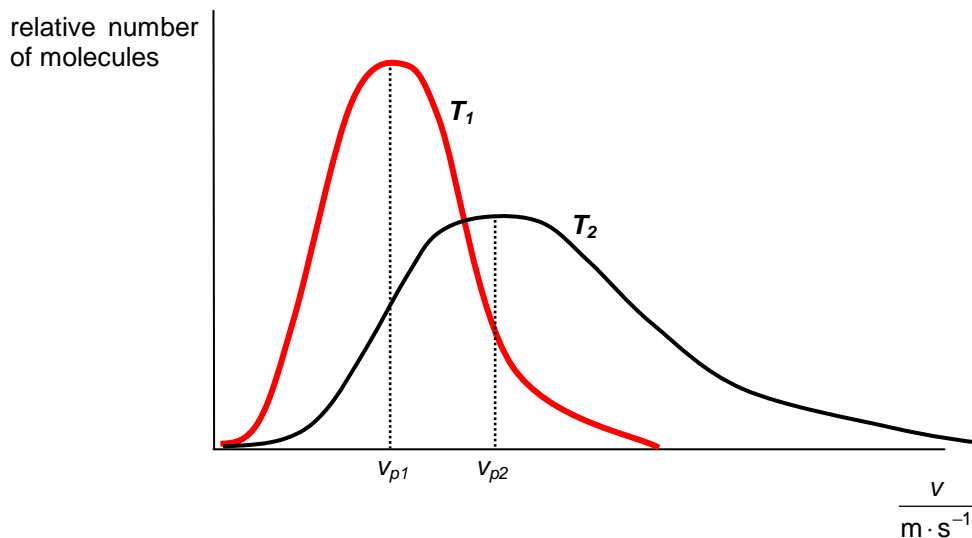
## 2. Speeds in a sample of gas

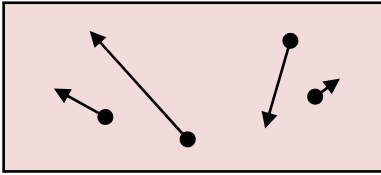
- the most probable speed  $v_p$  – most molecules have it
- average speed – not  $v_p$

### Root-mean-square speed (r.m.s. speed), $v_k$

-neither of the two previous ones

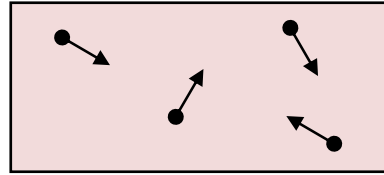
-when all of the molecules have the same speed – r.m.s., the kinetic energy of the sample is the same as when the molecules have different speeds (due to the collisions etc.)





$$p, V, T, N, v_1, v_2, \dots$$

$$E_k = \frac{1}{2} m \sum_{i=1}^N v_i^2$$



$$p, V, T, N, v_k$$

$$E_k = N \frac{1}{2} m v_k^2$$

$$v_k = \sqrt{\frac{\sum_{i=1}^N v_i^2}{N}}$$

### 3. Mean kinetic energy of a molecule

$$E_0 = \frac{1}{2} m_0 v_k^2 = \frac{3}{2} kT \quad k = 1.38 \times 10^{-23} \text{ J} \cdot \text{K}^{-1} \dots \text{ Boltzmann constant}$$

mass of ONE molecule

Mean kinetic energy of a molecule depends only on the temperature of the gas.

Kinetic energy of the sample of gas containing  $N$  molecules  $E = NE_0 = \dots\dots\dots$

$$R = kN_A = 8.31 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \dots \text{ universal molar gas constant}$$

$$\text{Avogadro number} = 6.023 \times 10^{23} \text{ mol}^{-1}$$

$$\text{number of moles} \quad n = \frac{m}{M_m} = \frac{N}{N_A}$$

#### Questions:

- Two samples of different gases (oxygen, nitrogen) have the same temperature. a) What is the relation between the mean kinetic energy of their molecules? b) What is the relation between the r.m.s. speeds of their molecules? c) When held in the same containers (volume) at the same pressure, what can you say about the number of particles of the gases?
- Calculate the r.m.s. speed of oxygen molecules at  $-100^\circ\text{C}$ ,  $0^\circ\text{C}$ ,  $100^\circ\text{C}$ .
- 100 g of argon have a temperature of  $20^\circ\text{C}$ . Calculate the total kinetic energy of the molecules in the sample.

#### 4. Pressure of a gas

is created because of the collisions of the molecules with the walls of the container

$$p = \frac{1}{3} \frac{N}{V} m_0 v_k^2 = \frac{1}{3} \rho v_k^2$$

L3/64-76

#### 5. Equation of state for an ideal gas

$$p = \frac{1}{3} \frac{N}{V} m_0 v_k^2 \quad v_k = \sqrt{\frac{3kT}{m_0}}$$

derive the equation below:

$$pV = nRT \quad \text{where} \quad n = \frac{m}{M_m} = \frac{N}{N_A}$$

This equation is valid only for low pressures and high temperatures. Van der Waals improved it – Nobel prize 1910.

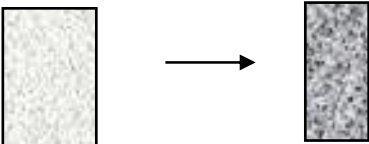
#### Avogadro's Principle (1811)

Two samples of different gases held at the same pressure, temperature and volume must contain the same number of molecules. (explain using the equation of state)

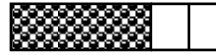
#### 6. Processes with an ideal gas

One sample of a gas – the number of moles (molecules, mass) does not change. One of the quantities  $p$ ,  $V$ ,  $T$  does not change, the other two change slowly – isothermal (IT), isovolumetric (IV) and isobaric (IB) process.

general eqn, valid even when ALL of them change:

$$\frac{p_1 V_1}{T_1} = nR = \frac{p_2 V_2}{T_2} = \text{const}$$


## 7. Isothermal process

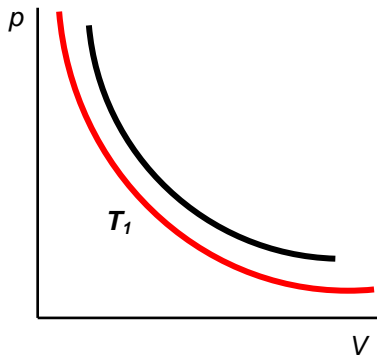


$$T_1 = T_2 = \text{const} \Rightarrow p_1 V_1 = p_2 V_2$$

$$pV = \text{const} \quad \text{Boyle's Law (Boylův-Mariottův zákon, IE, 1627-1691, F, 1620-1684)}$$

What does the „constant“ from B. law represent? How would you show IT process?

p-V diagram



label the axes and finish the diagrams:



## 8. Isovolumetric (isochoric) process

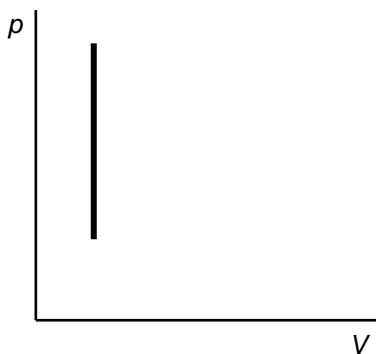
$$V_1 = V_2 = \text{const} \Rightarrow \frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$\frac{p}{T} = \text{const} \quad \text{Pressure Law (Charlesův zákon, F, 1746-1823)}$$

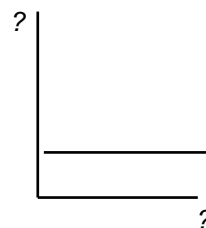
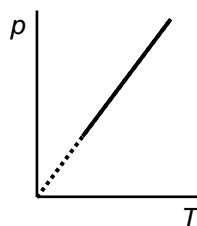
What does the „constant“ from the law represent?

Find the link between the use of a pressure cooker and this law. What is the difference?

p-V diagram



label the axes



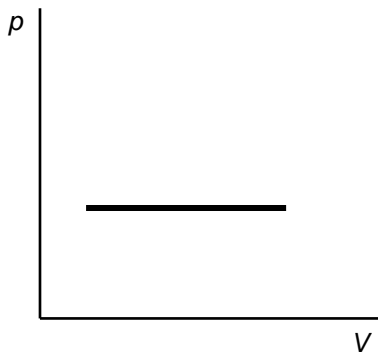
## 9. Isobaric process

$$p_1 = p_2 = \text{const} \Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

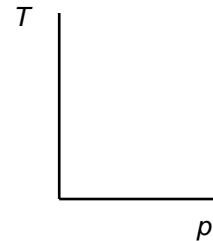
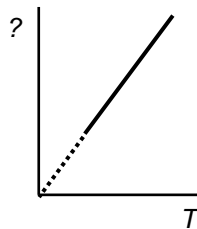
$$\frac{V}{T} = \text{const} \quad \text{Charles' Law (F, Gay-Lussacův zákon 1778-1850)}$$

What does the „constant“ from the law represent? Sketch the figure of the experiment seen and explain it.

p-V diagram



label the axis



L3/77-83, 86-7, X88-9, 90, X91-3

## 10. 1<sup>st</sup> Law of Thermodynamics and „iso-“ processes

- 1<sup>st</sup> Law of TD :  $Q = \Delta U + W$

*Heat supplied to a gas may raise its internal energy or enable it to expand.*

**Sign conventions:**

Q + when heat taken in

Q – when heat given out (cooled)

$\Delta U$  + when temperature rises

$\Delta U$  – when temperature decreases

W + when gas expands (V increases),

W – when compressed (V decreases)

- application on processes

a) IT

$$T = \text{const} \Rightarrow \Delta U = 0 \Rightarrow Q_T = W$$

BUT:

**2<sup>nd</sup> law of TD:** No continually working heat engine can take in heat from the source and convert it completely into work

Find and explain the difference between the two previous statements.

**b) IV**

$$V = \text{const} \Rightarrow W = 0 \Rightarrow Q_V = \Delta U$$

$$Q_V = mc_V \Delta t$$

$c_V$  ... specific heat capacity for isovolumetric process

**c) IB**

$$p = \text{const} \Rightarrow Q_p = \Delta U + W$$

$$Q_p = mc_p \Delta t$$

$c_p$  ... specific heat capacity for isobaric process

**Questions:**

4. Define the specific heat capacities for IV and IB processes. Can we find them anywhere?
5. What is the difference between them?
6. Is any of them always bigger no matter which gas it is for? Explain why (the following table is not sufficient).

GAS	$\frac{c_p}{\text{kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}}$	$\frac{c_p}{c_V}$
nitrogen	1.037	1.404
oxygen	0.912	1.401
hydrogen	14.189	1.41

## 11. Adiabatic process

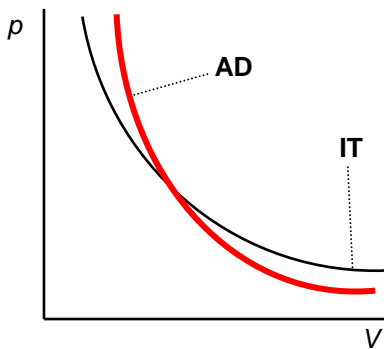
- all the quantities  $p, V, T$  change
  - quick compression/expansion
  - no  $Q$  exchanged (too quick)
- } differences between AD process and IT, IV and IB processes

$$pV^\kappa = \text{const} \quad \text{Poisson's Law (F, 1781-1840)}$$

$$\kappa \dots \text{kappa} \dots \text{Poisson's constant} \dots \kappa = \frac{c_p}{c_v} - 1$$

fill in the symbol equal/bigger/smaller

$\kappa$  is similar for gases having the same number of atoms in a molecule  
(1.4 for 2atomic, 1.66 for monatomic, 1.3 for 3atomic)



- examples of AD compression : Diesel engine (explain) [www.engines.ic.cz](http://www.engines.ic.cz)
- examples of AD expansion :  $\text{CO}_2$  fire extinguisher [http://en.wikipedia.org/wiki/Fire\\_extinguisher](http://en.wikipedia.org/wiki/Fire_extinguisher)  
 $\text{N}_2$  wart removal [http://www.medicinenet.com/warts\\_common\\_warts/article.htm](http://www.medicinenet.com/warts_common_warts/article.htm)

- 1<sup>st</sup> law of TD and AD process:

$$Q = 0 \Rightarrow 0 = \Delta U + W$$

Is the eqn  $\frac{pV}{T} = \text{const}$  also valid for AD process? Explain.

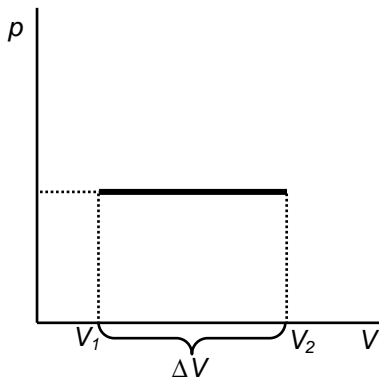
L3/ 94-95, 51-54

## 12. Work done by an ideal gas

- IB process

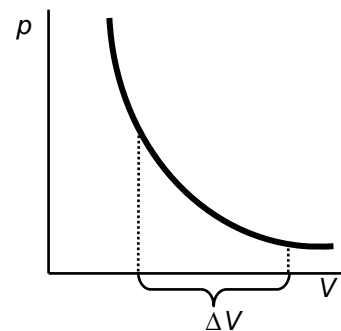
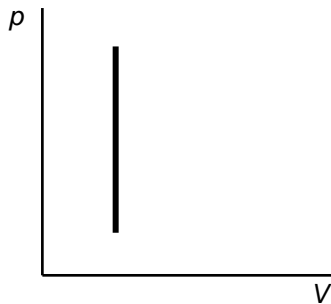
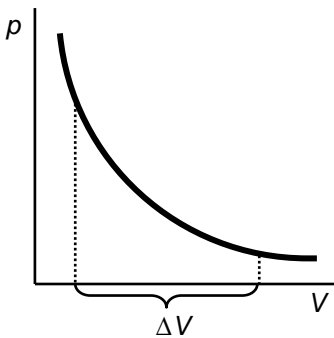


$$W = Fs = pAs = p\Delta V$$



area under p-V diagram = work done (for ANY shape/process!!!)

- other processes



What are the processes in the graphs above?

$$W = \int_{V_1}^{V_2} p dV$$



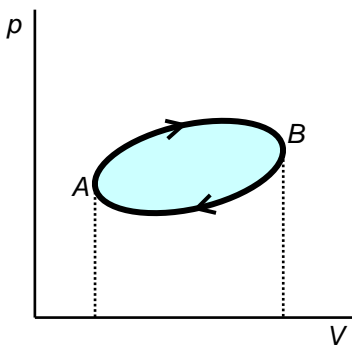
**Questions:**

7. How much work is done when 1.3 g of air at constant pressure changes its temperature from 20 °C to 100 °C? Take the molar mass of air 29 g·mol<sup>-1</sup>.

L3/97-104

### 13. Cycle with an ideal gas

When the sample of gas does some work, is compressed, heated, cooled and after a set of processes all the quantities ( $p$ ,  $V$ ,  $T$ ) are the same as they were at the beginning

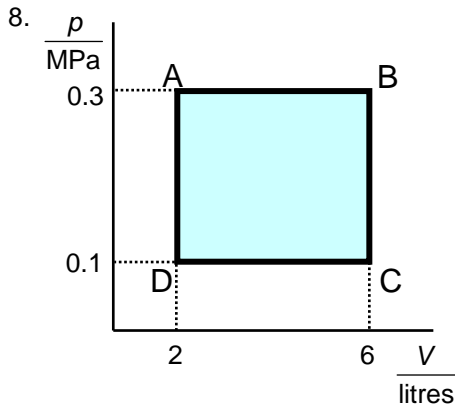


area under 1 –  $W$  done BY the gas (expansion)  
area under 2 –  $W$  done ON the gas (compression)

⇒

*pure work done by the gas during one cycle =  
area inside the curve/loop*

**Questions:**



- i) Which processes are represented by AB, BC, CD, DA?
- ii) Write eqn. of state and 1<sup>st</sup> law of TD
- iii) How much work is done during AB, BC, CD, DA?
- iv) Is the heat taken in/given out?
- v) Does the temperature rise/decrease?
- vi) How can we make the parts of the cycle?
- vii) What is the total work done during the cycle?

	i) which	ii) eqns	iii) W	iv) Q in/out	v) T up/down	vi) how to make
AB						
BC						
CD						
DA						

0.3

• **efficiency ( $\eta$ )**

$$\eta = \frac{\text{energy(power) output}}{\text{energy(power) input}} < 1$$

for one cycle :  $p, V, T$  the same  $\Rightarrow \Delta U = 0$

$$\Rightarrow W = Q = Q_1^{\text{in}} - Q_2^{\text{out}} = \text{energy output}$$

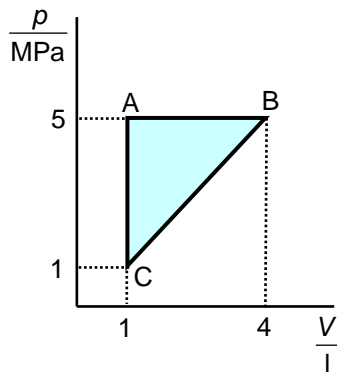
$Q_1 = \text{energy input}$

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} < 1$$

**Questions:**

9. Assume the heat taken in by the gas during one cycle 7 MJ and the heat given out during the cycle 3 MJ. How much work is done during the cycle and what is the efficiency of the process?

10. State the work done during AB, during BC, during CA and define the processes.



- **the Carnot cycle**

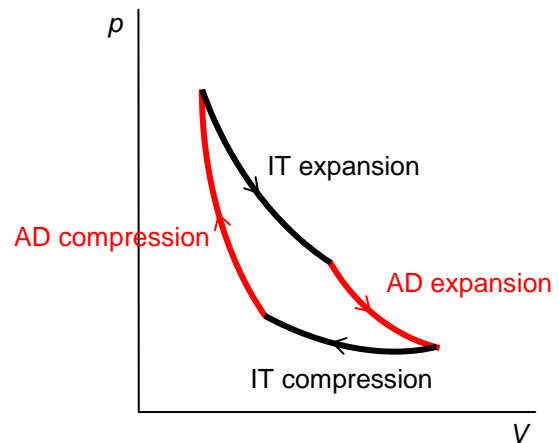
for an ideal cycle ... 2x AD + 2x IT processes

easy to measure temperatures, difficult to calculate heat

$T_1$  ... temperature of the source

$T_2$  ... temperature of the sink

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$



L3/105-110

## 14. Heat engines

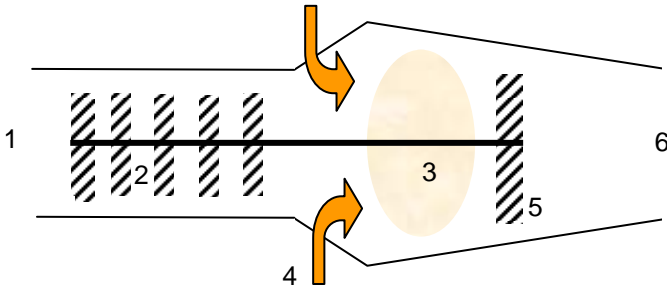
A part of the internal energy of the fuel burned is converted into work.

- **steam engines** – heat from the fuel burnt is used to heat water to get steam – high pressure steam – turbine rotates/piston moves – steam cooled – condensation – water – heated again = a cycle (James Watt, Scottish engineer, 1784 – improvement – the same liquid used)

- **combustion engines - petrol (spark) and Diesel engines (no spark)** – fuel + air – explosion – piston moves  
[www.engines.ic.cz](http://www.engines.ic.cz)

Use data from the internet to find the differences between two stroke and four stroke engines and explain their function.

- **jet engines** – similar fuel, work on the momentum conservation



1 – air sucked, 2 – compression of the air (more  $O_2$ ), 3 – space where fuel is burnt, 4 – jet supplying petrol  
5 – turbine for the compressor, 6 – jet

**Questions:**

- Find similarities and differences between the heat engines mentioned above.
- In which stroke of the four (two) stroke engine does the fuel work?

**Answers:**

- a) the same      b) heavier molecules are slower      c) the same
- a)  $367 \text{ m}\cdot\text{s}^{-1}$       b)  $461 \text{ m}\cdot\text{s}^{-1}$       c)  $539 \text{ m}\cdot\text{s}^{-1}$
- 9 kJ
- 29.8 J
- 4 MJ, 57%
- 15 kJ, (-)9 kJ, 0