INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

SIMPLE HARMONIC MOTION

1. Periodic motion, kinematics of simple harmonic motion

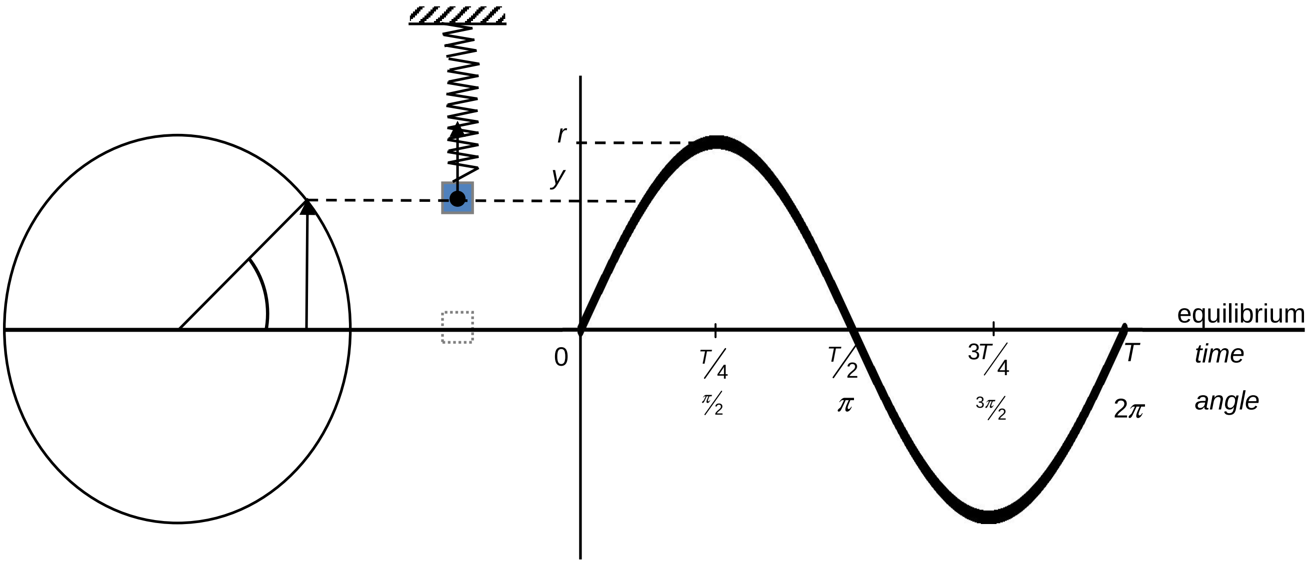
- motion – linear, circular, „to-and-fro“ = vibration/oscillation
- oscillation can be irregular, but we will discuss just the regular one called SIMPLE HARMONIC MOTION which
  - is connected with a steady circular motion ($r = y_{max}, T$)
  - can be described by the eqn: $a = -ky$

acceleration and displacement from equilibrium at the same time instant

2. Quantities that describe s.h.m., phasor diagrams

http://fyzika.jreichl.com/index.php?sekce=browse&page=156

a) displacement



$$y = r \sin \omega t = y_m \sin \omega t$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Questions:

1. A mass suspended on a spring provides a s.h.m. of period 2 seconds. Assume the maximum displacement from equilibrium 10 cm and the beginning of the measurement when the object moves up passing the equilibrium position; calculate the displacement of the object after one second and 23 seconds.

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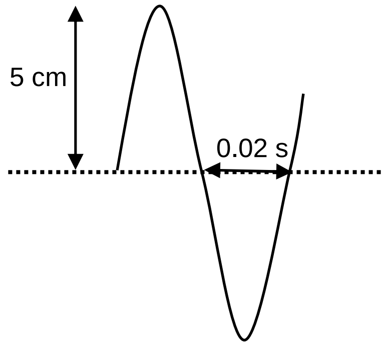
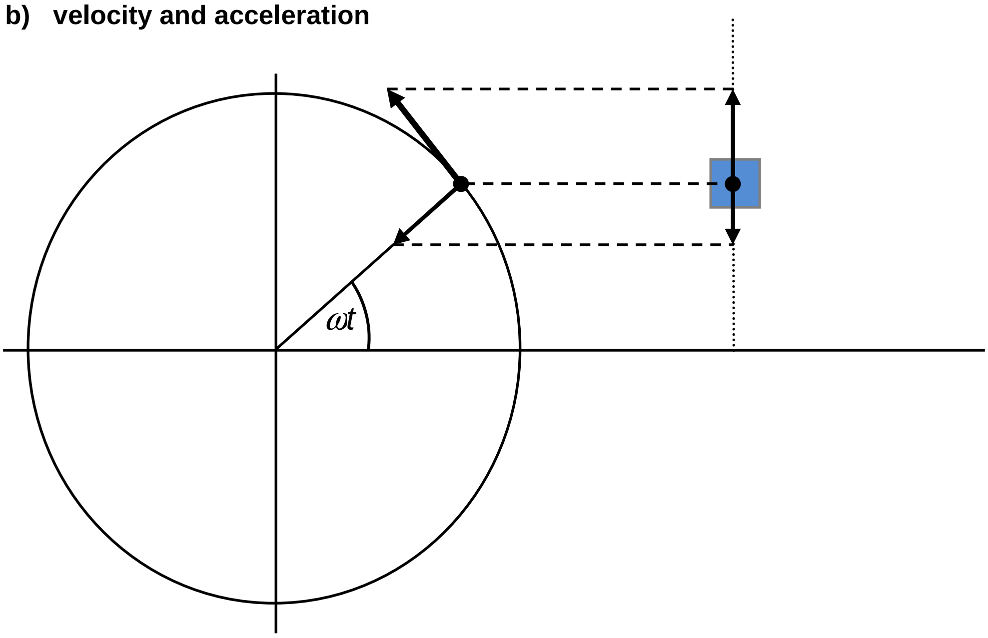
2. Write the eqn. $y = f(t)$



b) velocity and acceleration



velocity and acceleration of the object performing a s.h.m. are the side projections of the similar quantities connected with the „suitable" s.c.m.

$v = v_0 \cos \omega t = \omega r \cos \omega t = \omega y_m \cos \omega t$

size of $a = a_c \sin \omega t = \omega^2 r \sin \omega t = \omega^2 y_m \sin \omega t$, but opposite direction to the displacement – so finally

$a = -\omega^2 y_m \sin \omega t = -\omega^2 y$

The equations can be derived as the first and second derivative of the displacement according to the time

$v = y' = \frac{dy}{dt} =$

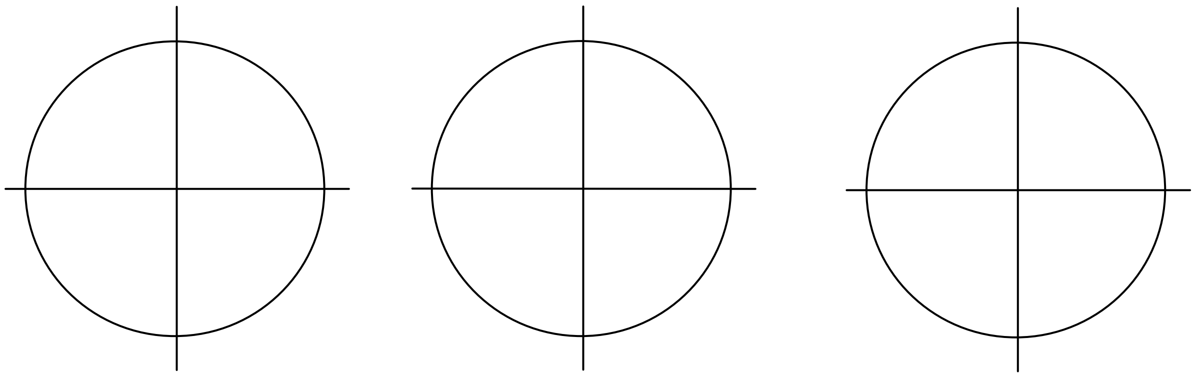
$a = v' = \frac{dv}{dt} = \frac{d^2 y}{dt^2} =$

advantage – simple, we can work out +/- from the eqns itself
disadvantage – „just maths", for everyone not aware of the connection with reality

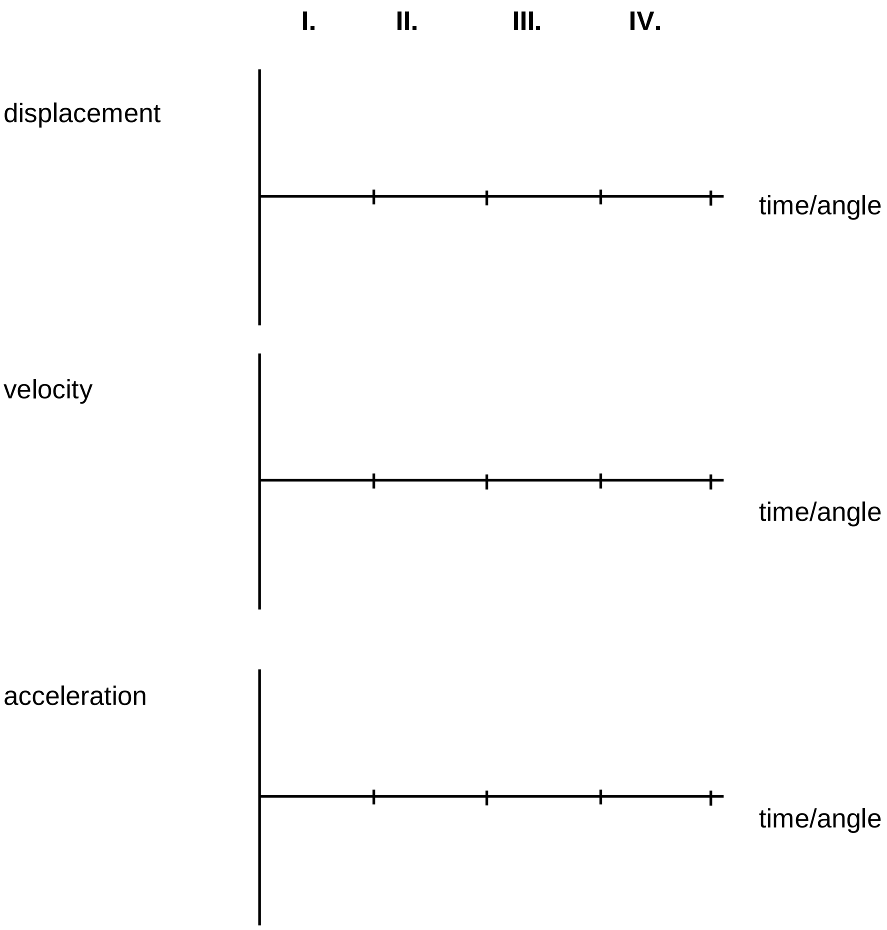
Questions:

3. Compare with the first definition eqn. and discuss.

4. Sketch similar figures for quadrants II. – IV. Relate the direction of the velocity with the direction of motion. State from the mutual direction of the velocity and acceleration if the motion is accelerated or decelerated. Find the points with the maximum values of velocity (acceleration) and zero values of the same quantities.

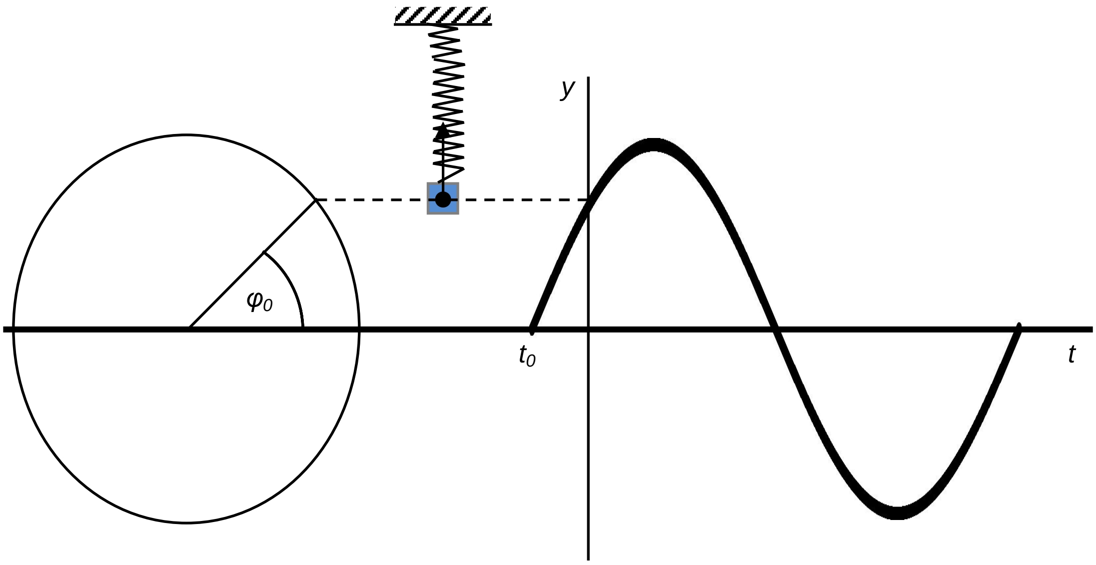


c) y-t, v-t, a-t graphs



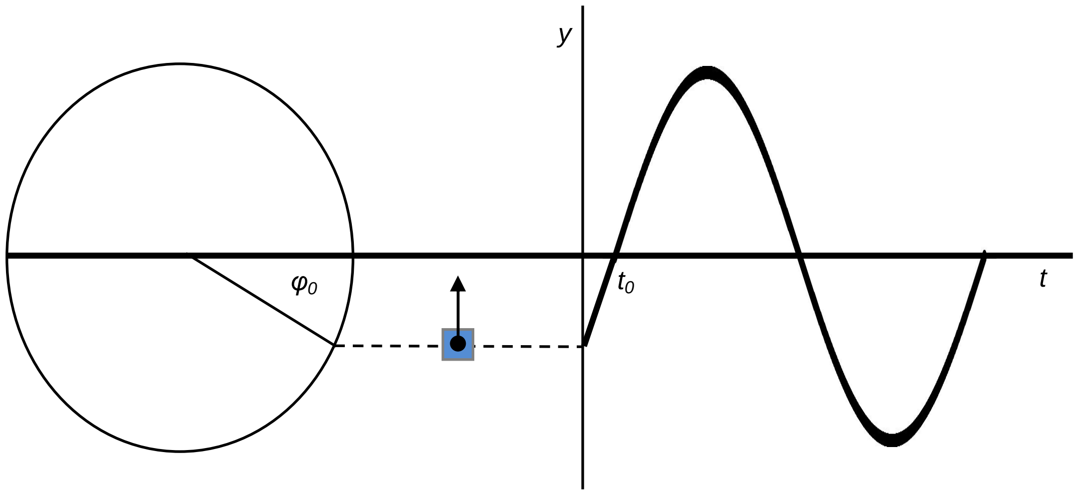
3. Initial phase

- important when we do not start to measure the time when the object passes the equilibrium position „up“
- appropriate angle/time should be added or subtracted to get the complete sine or cosine curve



$\varphi_0 = \omega t_0$ $y = y_m \sin(\omega t + \varphi_0)$ *v* = *a* =

when the angle is from π to 2 π , initial phase can be subtracted:



$y = y_m \sin(\omega t - \varphi_0)$ *v* = *a* =

Questions:

5. Calculate the initial phase when

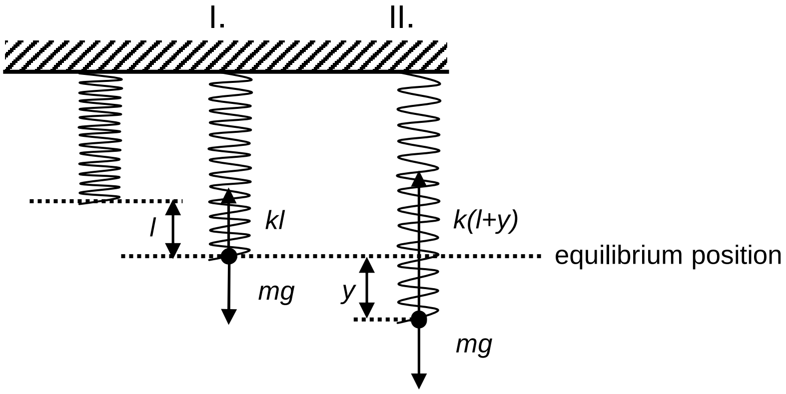
a) t_0 = 0.5 s, T = 4 s
b) t_0 = 0.1 s, T = 2 s

L4/1-14, X15, 16-23

4. Mass on a spring

- springs obey Hooke's law
- *k* ... spring constant = the force needed to produce unit extension (=1m)

$k = \frac{F}{l}$ $\qquad [k] = \mathrm{N\,m^{-1}}$



I. $F_R = 0$

$F_S = F_G$

$kl = mg$

II. $F_R \neq 0$

$F_R = F_S - F_G$

$F_R = k(l + y) - mg = kl + ky - kl = ky$

The resultant causes acceleration according to Newton's 2nd Law

sizes: $ma = ky$, but acceleration and displacement have opposite directions, so:

$$ma = -ky$$
$$a = -\frac{k}{m}y$$

compare: $a = -\omega^2 y$ hence

$$\omega^2 = \frac{k}{m} \qquad \text{take } \omega = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{or} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

T, f ... natural period, frequency

Questions:

6. A light spiral spring is loaded with a mass of 50 g and it extends by 10 cm. Assume the maximum displacement from equilibrium of 5 cm, zero initial phase and calculate:

a) the period of small vertical oscillations
b) the velocity at equilibrium
c) acceleration 2 cm above equilibrium
d) the time taken to get 2 cm above equilibrium

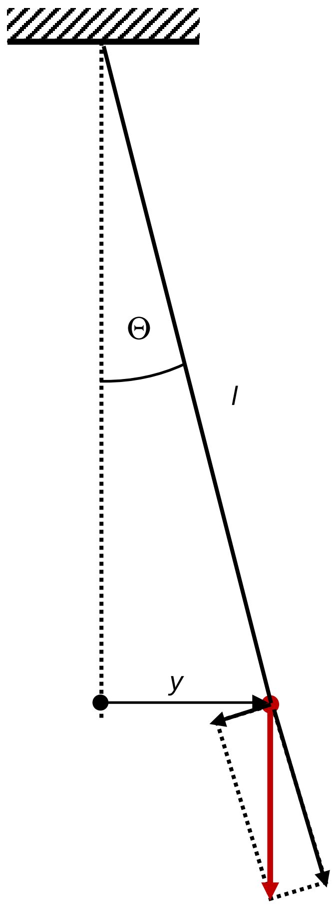
5. Simple pendulum

- a small bob of mass *m* suspended by a light inextensible string of length *l* from a fixed point
- for small angle only (less than 5 degrees), when the displacement can be almost a straight line
- weight of the bob ... *mg*

  $mg\cos\Theta$ ... its „tension" component – balanced by the string force

  $mg\sin\Theta$ ... its tangential component ="restoring force", unbalanced – 2nd NL!!!

Find these forces in the following figure:



$$mg\sin\Theta = ma$$

$$mg\frac{y}{l} = ma$$

$$a = \frac{g}{l}y$$ for the sizes, but acceleration and displacement have opposite directions, so:

$$a = -\frac{g}{l}y$$

compare $a = -\omega^2 y$ hence

$\omega^2 = \frac{g}{l}$ take

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{or} \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

T, f ... natural period, frequency

Questions:

7. Explain why the period of a simple pendulum does NOT depend on the mass of the load. Prove it!

8. A ball of mass 40 g hangs on a string 50 cm long. Then it is pushed and it starts to move to-and-fro having maximum distance from equilibrium position 2 cm. When we start to measure the time at maximum displacement, calculate:

a) the frequency of the oscillations
b) time taken to get to equilibrium
c) the displacement when the speed is just half of the maximum one
d) sketch the displacement-time graph

L4/29-34, 42-51

6. Dynamics of s.h.m.

- s.h.m. is accelerated $a = -\omega^2 y$
- the acceleration is caused by the resultant force acting on the oscillating object (2nd N.L.)

$F = ma = -m\omega^2 y$ rises with the displacement but it is always pointed towards equilibrium!

Questions:

9. Discuss *F* in both previous examples of s.h.m.
   a) Which forces act on the mass on a spring (simple pendulum) and what is their resultant?
   b) How is the size of the resultant related to the displacement?

10. Look at all of the previous calculations and if you have enough information, calculate the maximum resultant force and the force at particular displacements from the examples. If you cannot do that, state which data are missing.

7. Energy of s.h.m.

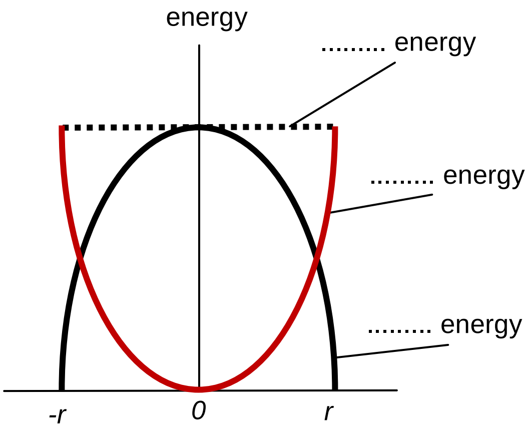
- assume FREE (undamped) oscillation, where the mechanical energy is not converted into other types
- the total mechanical energy remains constant, only the values of potential and kinetic energy can change to obey the formula

$E_{mech} = E_k + E_p$

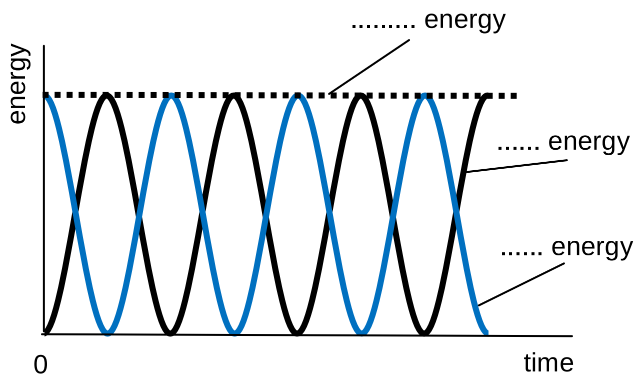
- **at equilibrium** - kinetic energy is..................... (the object has the maximum speed), potential energy is zero
- **at maximum displacement** – kinetic energy is........... (the object stops there), potential energy is maximum

$$E_{k\max} = \frac{1}{2}mv_0^2 = \frac{1}{2}m\omega^2 r^2 = E_{p\max} = E_{mech}$$

energy – displacement graph



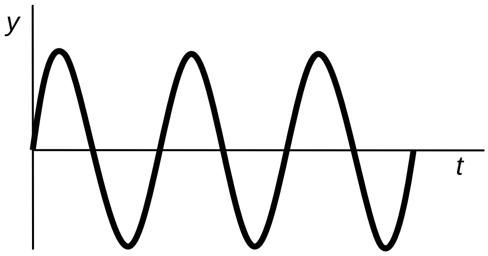
energy – time graph



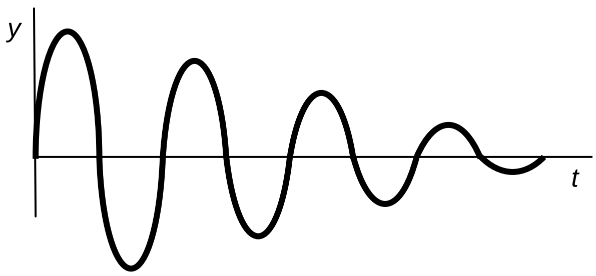
On the time axis label the fractions and multiples of period (assume zero initial phase)

8. Free, damped and forced oscillations, resonance

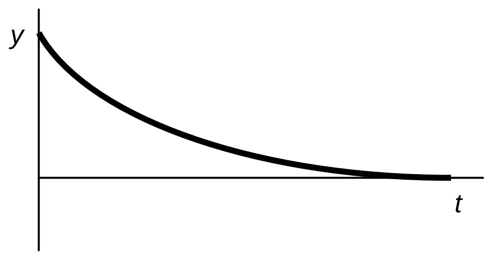
- **free oscillations** – mechanical energy is conserved – ideal situation only

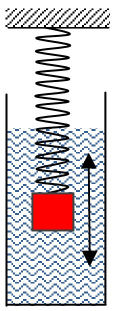


- **damped oscillation –** mechanical energy is converted into other types – real situation
  - the amplitude of the oscillations gradually decreases, THE PERIOD STAYS THE SAME (see the eqn)
    http://www.lon-capa.org/~mmp/applist/damped/d.htm
    http://paws.kettering.edu/~drussell/Demos/SHO/damp.html
  - lightly damped – pendulum in the air



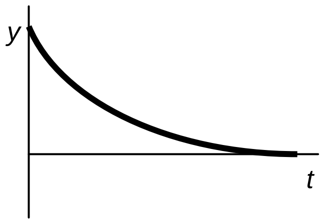
- heavily damped – no oscillation, the object just returns to equilibrium





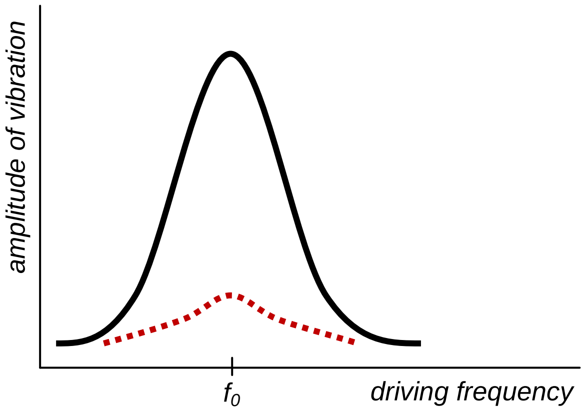
- critically damped = heavily damped during the shortest possible time T/4 – shock absorbers

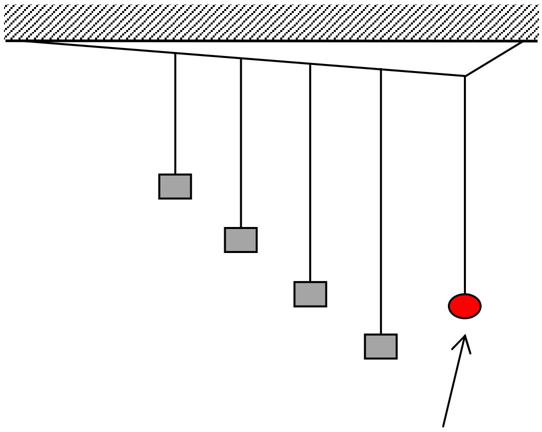
http://en.wikipedia.org/wiki/Shock_absorber
http://auto.howstuffworks.com/car-suspension2.htm



- **forced oscillations** – real situation where an external force is used to keep the oscillations

e.g. a swing or Barton's pendulums – the force applied is not the only important quantity, the force should be applied in suitable time intervals (driving frequency) to do minimum work (min. energy needed to keep oscillations) = **resonance**





Questions:

11. Look at all of the previous calculations and if you have enough information, calculate the total mechanical energy stored in the oscillating system, maximum kinetic and maximum potential energy. Assume free oscillations. If you cannot do that, state which data are missing.

L4/66

Answers:

1. a) 0 b) 0
2. 0.05sin(157t)
5. a) 0.25 rad = 45º b) 0.1 rad = 18º
6. a) 0.63 s; b) 0.5 $m \cdot s^{-1}$; c) 2 $m \cdot s^{-2}$; d) 0.04 s
8. a) 0.7 Hz; b) 0.36 s; c) ±1.732 cm
10. 6. 0.25 N; 0.1 N
    8. 15.5 mN; 13.4 mN
11. 6. 6.25 mJ
    8. 0.15 mJ